Principles of Computer Game Design and Implementation

Lecture 10

Quiz

We already learned

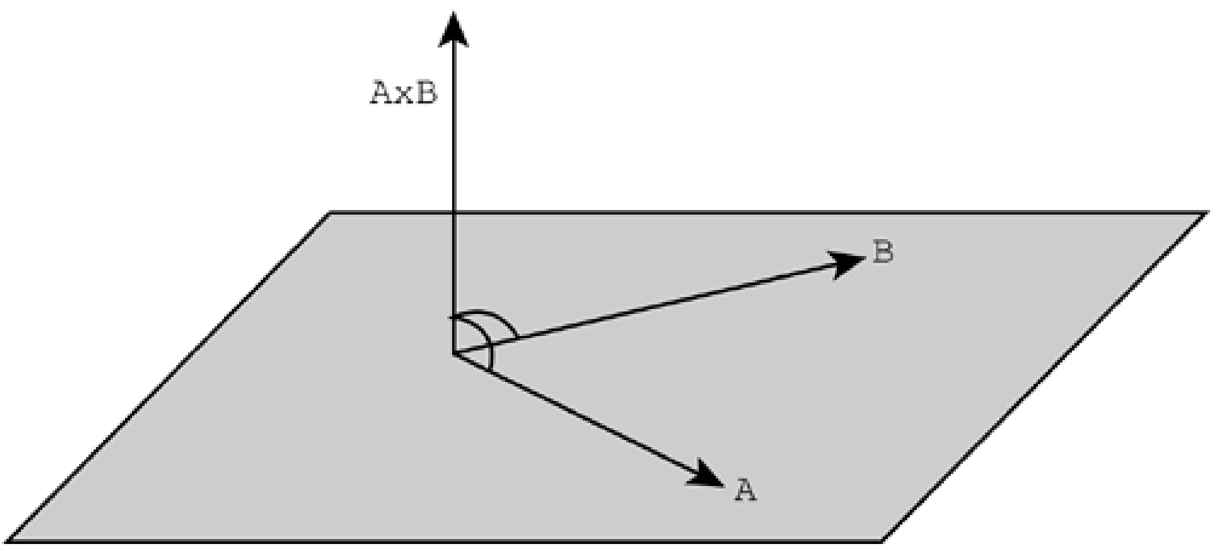
- Translation
- Movement
- Rotation
- Dot product

Outline for today

- Cross product
- Explanation of first assignment

The Cross Product

- The cross product of two vectors is a vector
  - Only applies in three dimensions
  - *The cross product is perpendicular to both vectors*
    - The cross product between two parallel vectors is the zero vector (0, 0, 0)



The Cross Product

- The cross product between **V** and **W** is

$$\mathbf{V} = \begin{pmatrix} x_v, & y_v, & z_v \end{pmatrix} \qquad \mathbf{W} = \begin{pmatrix} x_w, & y_w, & z_w \end{pmatrix}$$

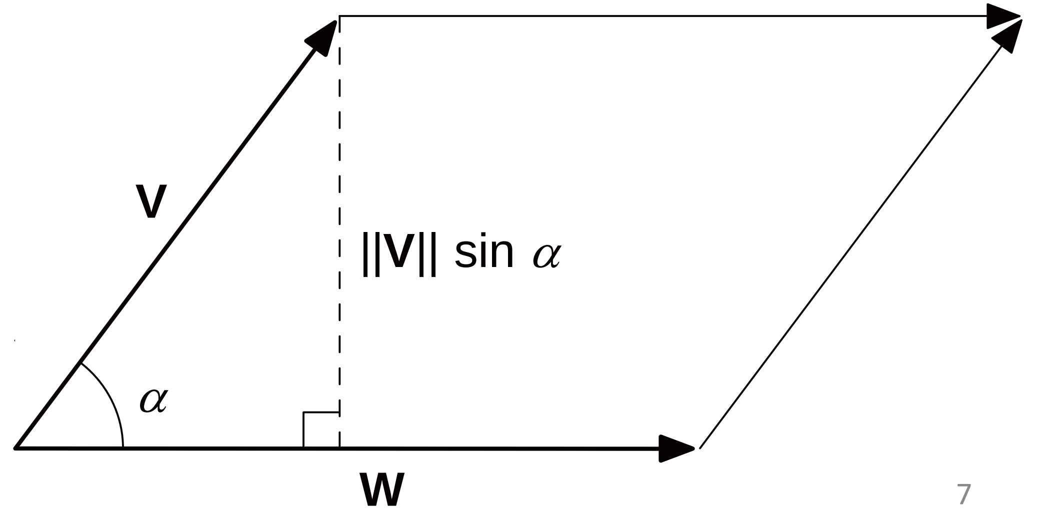
$$\mathbf{V} \times \mathbf{W} = \begin{pmatrix} y_v z_w - z_v y_w, & z_v x_w - x_v z_w, & x_v y_w - y_v x_w \end{pmatrix}$$

The Cross Product

- The cross product satisfies the trigonometric relationship

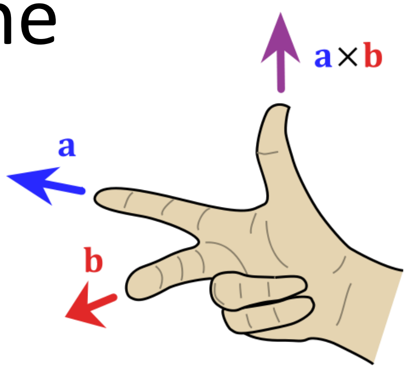
$$\|\mathbf{V} \times \mathbf{W}\| = \|\mathbf{V}\|\|\mathbf{W}\| \sin \alpha$$

- This is the area of the parallelogram formed by **V** and **W**



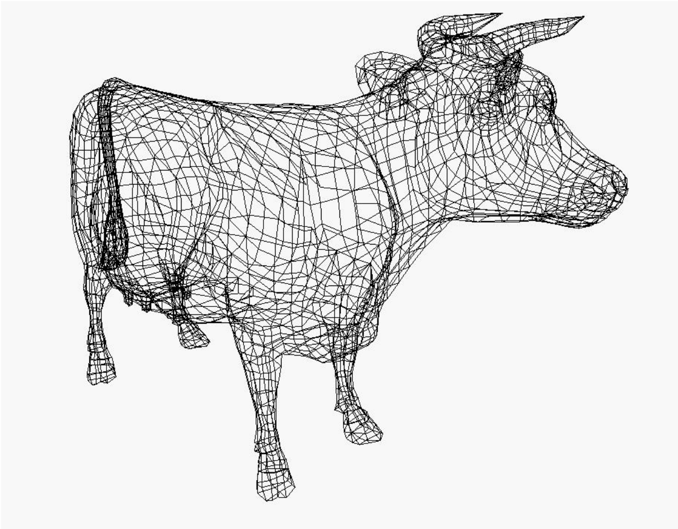
The Cross Product

- Cross products obey the right hand rule
  - If first vector points along right index finger, and second vector points along middle finger,
  - Then cross product points out of right thumb
- Reversing order of vectors negates the cross product:



$$\mathbf{W} \times \mathbf{V} = -\mathbf{V} \times \mathbf{W}$$

Uses: Face Normal

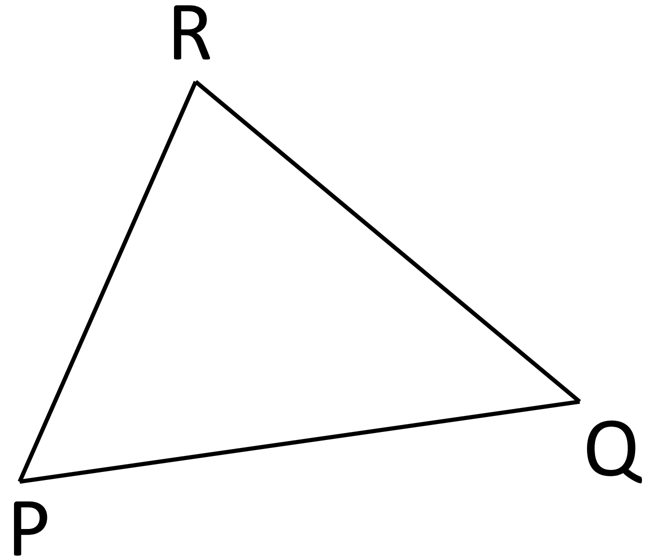


- Complex 3D models are build from *polygons* mostly *triangles*

- When determining the luminance of a triangle, we need to know the angle between the plain in which it lays and the light beam.

Example: Normal of a Triangle

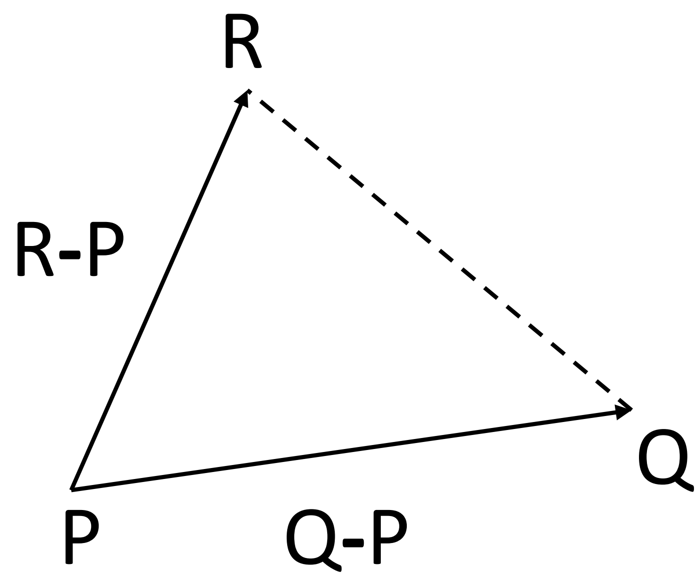
- Find the unit length normal of the triangle defined by 3D points P, Q, and R



Example: Normal of a Triangle

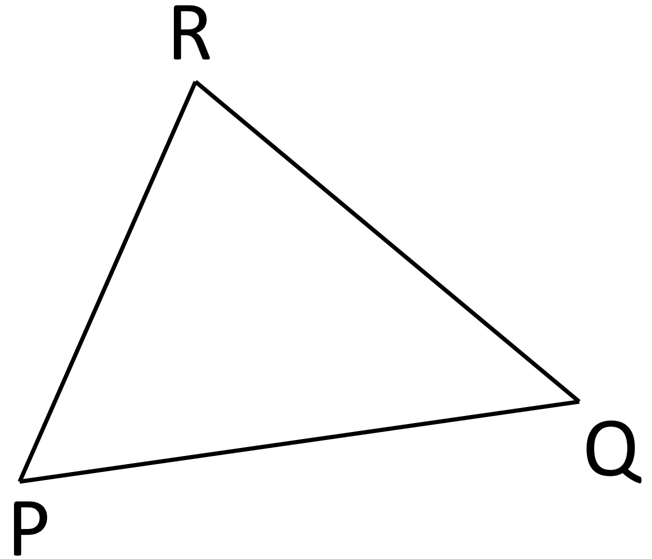
$$\mathbf{n}^{*} = (R - P) \times (Q - P)$$

$$\mathbf{n} = \frac{\mathbf{n}^{*}}{\left|\mathbf{n}^{*}\right|}$$



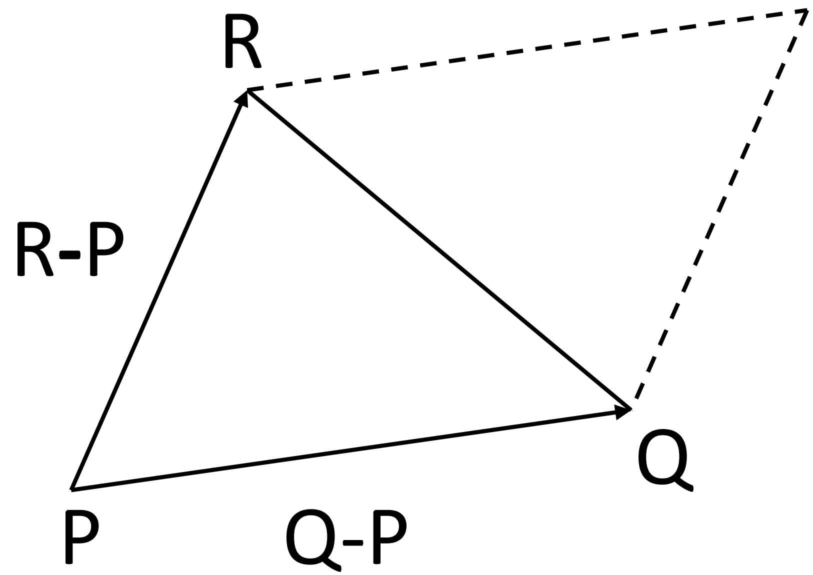
Example: Area of a Triangle

- Find the area of the triangle defined by 3D points P, Q, and R



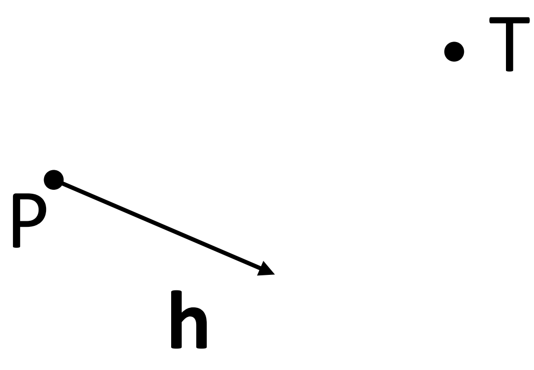
Example: Area of a Triangle

$$area = \frac{1}{2}\left|(Q - P) \times (R - P)\right|$$



Example: Alignment to Target

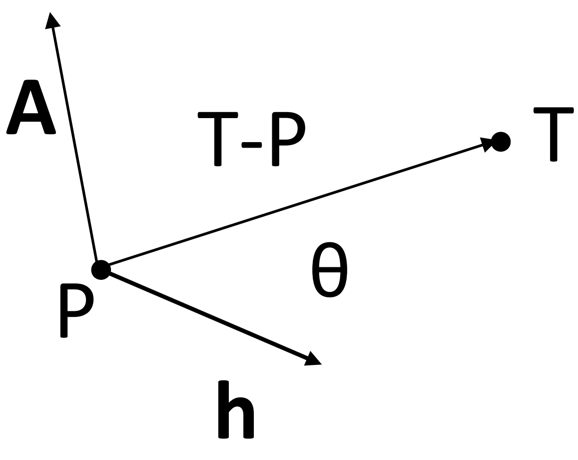
- An object is at position P with a unit length heading of **h**. We want to rotate it so that the heading is facing some target T. Find a unit axis **A** and an angle θ to rotate around.



Example: Alignment to Target

$$\mathbf{a} = \frac{\mathbf{h} \times (T - P)}{\left|\mathbf{h} \times (T - P)\right|}$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{h} \cdot (T - P)}{\left|(T - P)\right|}\right)$$



jME Example

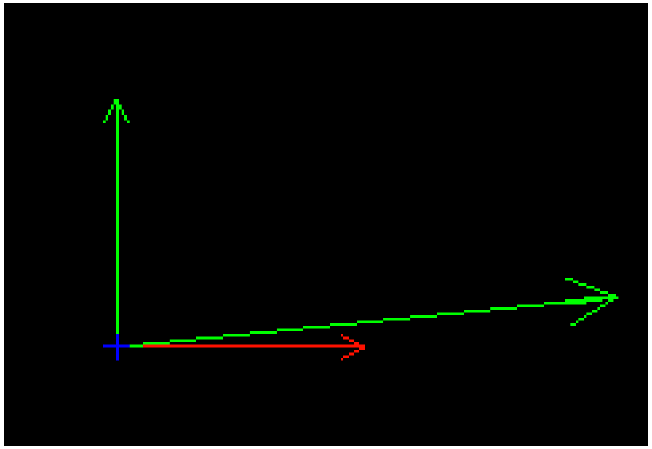
```
Vector3f u = new Vector3f(x, y, z).normalize();


Arrow yArrow = new Arrow(Vector3f.UNIT_Y);
gyArrow = new Geometry("Y", yArrow);
…
rootNode.attachChild(gyArrow);

Vector3f axis = Vector3f.UNIT_Y.cross(u);
float angle = FastMath.acos(Vector3f.UNIT_Y.dot(u));

Quaternion q = new Quaternion();
q.fromAngleAxis(angle, axis);
gyArrow.setLocalRotation(q);
```

It Works



(I've added the AxisRods to the picture to show the reference point)

Gradual Rotation: simpleUpdate

```
float curT = t.getTimeInSeconds();
if (curT < (startT + timeoutR))
{
   float currentAngle =
     (startAngle + ((curT-startT)/
                    timeoutR)*targetAngle);
   Quaternion q = new Quaternion();
   currentAngle += tpf;
   q.fromAngleAxis(currentAngle, axisV);
   gyArrow.setLocalRotation(q);
}
```

Conclusion

- Dot and cross products are both used in 3D graphics
- Dot product is a number
- Cross product is a vector

$$\mathbf{V} = \left(x_v, \quad y_v, \quad z_v \right) \qquad \mathbf{W} = \left(x_w, \quad y_w, \quad z_w \right)$$

$$\mathbf{V} \bullet \mathbf{W} = \left(x_v x_w + \quad y_v y_w + \quad z_v z_w \right)$$

$$\mathbf{V} \times \mathbf{W} = \left(y_v z_w - z_v y_w, \quad z_v x_w - x_v z_w, \quad x_v y_w - y_v x_w \right)$$