Parametric Linear Temporal Logics

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Motivation

Linear Temporal Logic (LTL) as specification language:

- Simple and variable-free syntax and intuitive semantics.
- Expressively equivalent to first-order logic on words.
- LTL model checking routinely applied in industrial settings.

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2. LTL cannot express all ω-regular properties.
   - Many extensions that are equivalent to ω-regular languages: add regular expression-, grammar-, or automata-operators to LTL.

Overview

LTL

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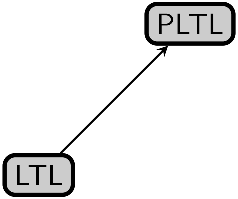

Parametric LTL

Alur et al. '99: add parameterized operators to LTL

$$\varphi ::= p \mid \neg p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \mathbf{X}\varphi \mid \varphi \mathbf{U} \varphi \mid \varphi \mathbf{R} \varphi \mid \mathbf{F}_{\leq x}\varphi \mid \mathbf{G}_{\leq y}\varphi$$

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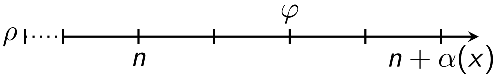
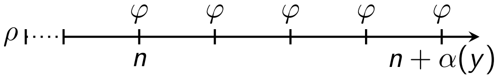
- $(\rho, n, \alpha) \models \mathbf{F}_{\leq x}\varphi$: 

- $(\rho, n, \alpha) \models \mathbf{G}_{\leq y}\varphi$: 

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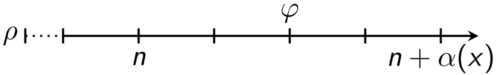
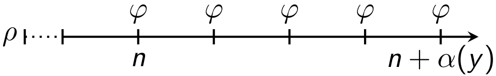
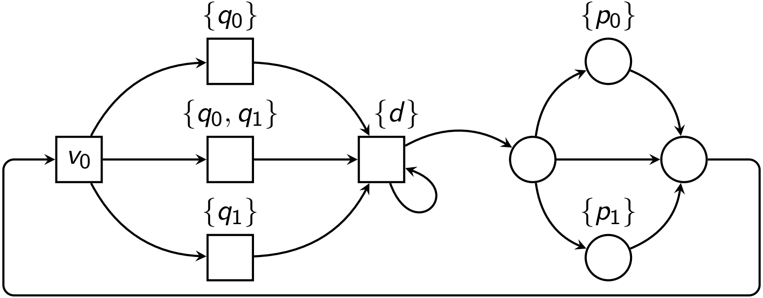
- $(\rho, n, \alpha) \models \mathbf{F}_{\leq x}\varphi$: 

- $(\rho, n, \alpha) \models \mathbf{G}_{\leq y}\varphi$: 

Fragments:

- $\text{PLTL}_{\mathbf{F}}$: no parameterized always operators $\mathbf{G}_{\leq y}$.
- $\text{PLTL}_{\mathbf{G}}$: no parameterized eventually operators $\mathbf{F}_{\leq x}$.

PLTL Games



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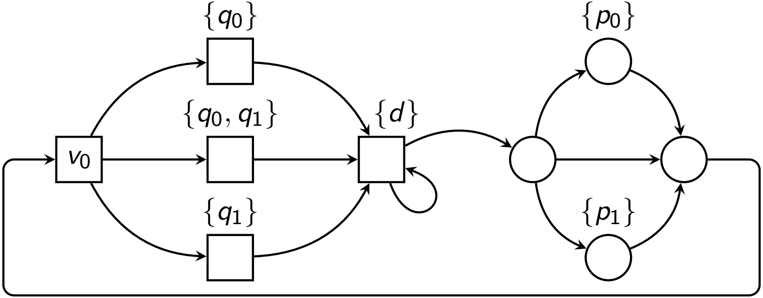

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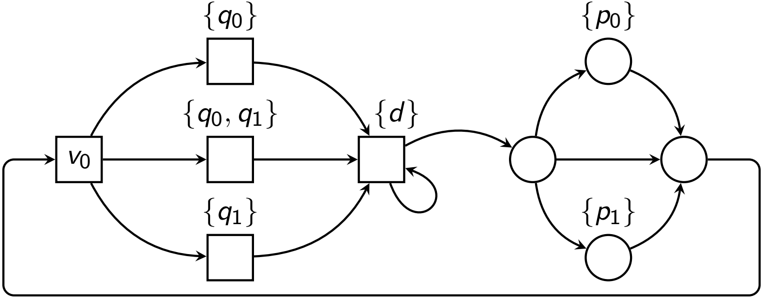

- $\varphi_1 = \mathbf{FG}d \vee \bigwedge_{i\in\{0,1\}} \mathbf{G}(q_i \rightarrow \mathbf{F}p_i)$: Player 0 wins.
- $\varphi_2 = \mathbf{FG}d \vee \bigwedge_{i\in\{0,1\}} \mathbf{G}(q_i \rightarrow \mathbf{F}_{\leq x_i}p_i)$: Player 1 wins w.r.t. every α.

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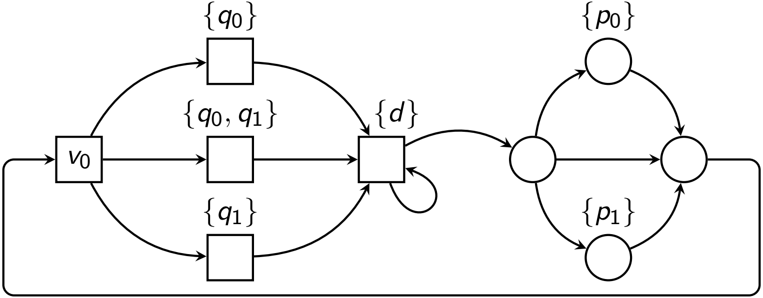

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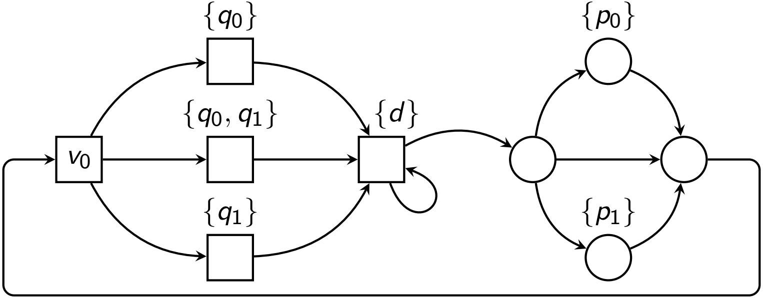

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Lemma (Determinacy)

$\mathcal{W}_0(\mathcal{G})$ is the complement of $\mathcal{W}_1(\mathcal{G})$.

Decision Problems

- Membership: given $\mathcal{G}$, $i \in \{0, 1\}$, and α, is $\alpha \in \mathcal{W}_i(\mathcal{G})$?
- Emptiness: given $\mathcal{G}$ and $i \in \{0, 1\}$, is $\mathcal{W}_i(\mathcal{G})$ empty?
- Finiteness: given $\mathcal{G}$ and $i \in \{0, 1\}$, is $\mathcal{W}_i(\mathcal{G})$ finite?
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Theorem (Pnueli, Rosner '89)

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Adding parameterized operators does not increase complexity:

Theorem (Z. '11)

All four decision problems are $2\textsc{Exptime}$*-complete.*

Proof Sketch (Emptiness)

1. Replacing $\mathbf{G}_{\leq y}\psi$ by ψ preserves emptiness (monotonicity).
2. Apply alternating color technique **(Kupferman et al. '06)**:
   - Add new proposition p and replace every $\mathbf{F}_{\leq x}\psi$ by

     $$(p \rightarrow p\mathbf{U}(\neg p\mathbf{U}\psi)) \wedge (\neg p \rightarrow \neg p\mathbf{U}(p\mathbf{U}\psi))$$

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3. Emptiness for game with condition φ equivalent to Player 0 winning LTL game with condition $c(\varphi) \wedge \mathbf{GF}p \wedge \mathbf{GF}\neg p$, as finite state strategies bound distance between color changes.
4. Yields doubly-exponential upper bound.

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- $\min_{\alpha \in \mathcal{W}_0(\mathcal{G}_{\mathsf{F}})} \max_{x \in \mathrm{var}(\varphi_{\mathsf{F}})} \alpha(x)$ for $\mathrm{PLTL}_{\mathsf{F}}$-formula φ_{F}.

1. Replacing every variable by z preserves optimum (monotonicity).
2. Doubly-exponential upper bound on optimum.
3. Models of φ w.r.t. α recognized by deterministic parity automaton of triply-exponential size, provided $\alpha(z)$ is at most doubly-exponential.
4. Thus, $\alpha \in \mathcal{W}_0(\mathcal{G}_{\mathsf{F}})$ can be decided in triply-exponential time.
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Note:
Doubly-exponential lower bound on optimum rules out doubly-exponential running time for this algorithm.

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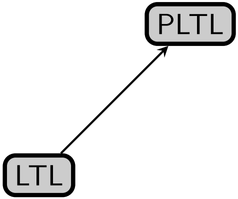

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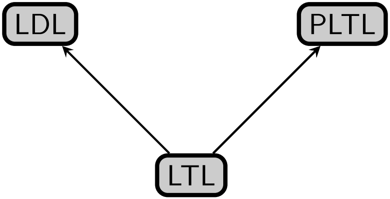

Linear Dynamic Logic

Vardi '11: Another extension of LTL expressing exactly the ω-regular languages: use PDL-like operators

$$\varphi ::= p \mid \neg p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \langle r \rangle \varphi \mid [r]\varphi$$

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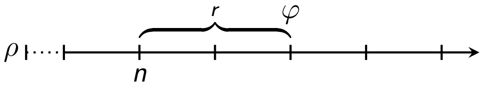

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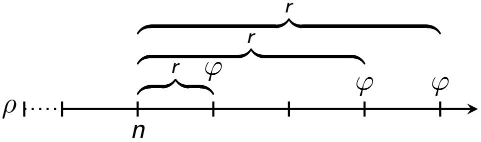

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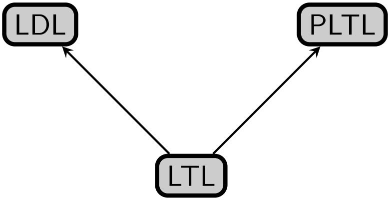

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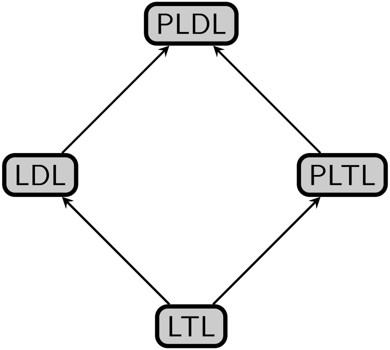

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We are interested in same decision problems as for PLTL

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as well as the optimization problems.

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   - Introduce color change aware operators: $\langle r\rangle_{\mathrm{cc}}$ only takes into account matches of r within at most one color change.
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Translating LDL_{cc} into Automata

Theorem (Faymonville, Z. '14)

LDL_{cc}-formulas can be translated into linearly-sized alternating automata.

Proof Sketch:

Bottom up construction:

- Atomic formulas, conjunction, and disjunction straightforward.

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- $[r]\psi$: as for $\langle r\rangle\psi$, but make all states of $\mathfrak{A}_r$ universal to test for all matches of r.
- $\langle r\rangle_{\mathrm{cc}}\psi$: as for $\langle r\rangle\psi$, but take intersection of $\mathfrak{A}_r$ and automaton checking for at most one color change.

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PLDL optimization problems are solvable in

- *polynomial space for model checking, and*
- *triply-exponential time for games.*

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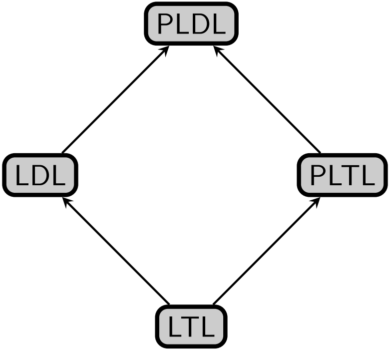

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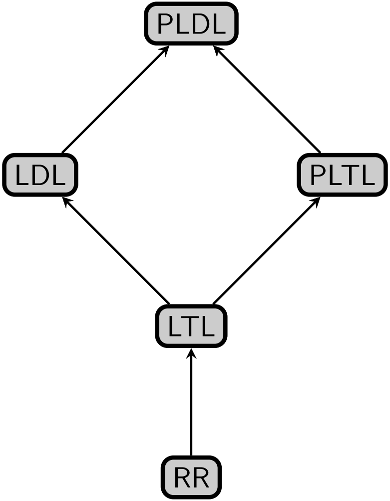

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Corollary

- *Finite-state winning strategies of size $k2^{k+1}$ for both players.*
- *Solvable in* EXPTIME.

Waiting Times

- $\mathrm{wt}_j(\varepsilon) = 0$, and

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Goal:
Prove that optimal winning strategies exist and are computable.

Main Theorem

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Optimal strategies for RR games exist, are effectively computable, and finite-state.

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2. Expand arena by keeping track of waiting time vectors up to bound from 1.). RR-values equal to mean-payoff condition.
   - Optimal strategy for mean-payoff yields optimal strategy for RR game.

Dickson's Lemma

Dickson pair: $((x_1, \ldots, x_k), (y_1, \ldots, y_k)) \in \mathbb{N}^k$ s.t. $x_j \leq y_j$ for all j.

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Waiting time vectors are special:

- either increment, or
- reset to zero.

Lemma

There is a function $b(|\mathcal{A}|, k) \in \mathcal{O}(2^{2^{|\mathcal{A}| \cdot k + 2}})$ such that every play infix of length $b(|\mathcal{A}|, k)$ has a dickson pair.

Bounding the Waiting Times

We have σ with $\mathrm{val}(\sigma, v) \leq \sum_{j=1,\ldots,k} |\mathcal{A}| k 2^k =: b_{\mathcal{G}}$ for all $v \in W_0(\mathcal{G})$.

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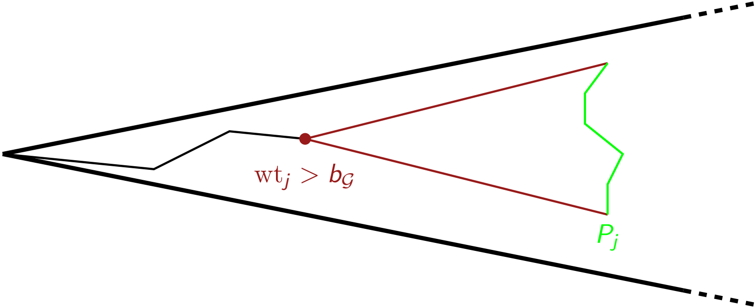

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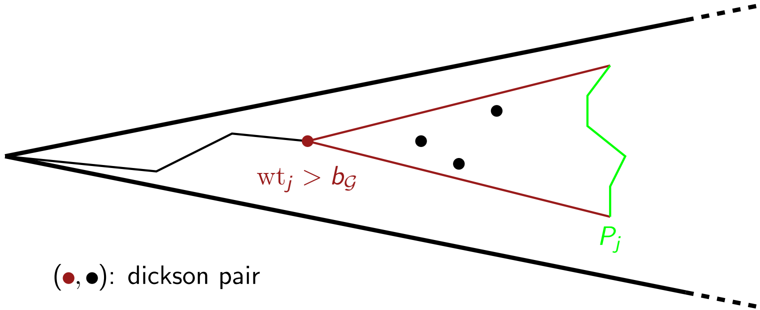

($\bullet$, $\bullet$): dickson pair

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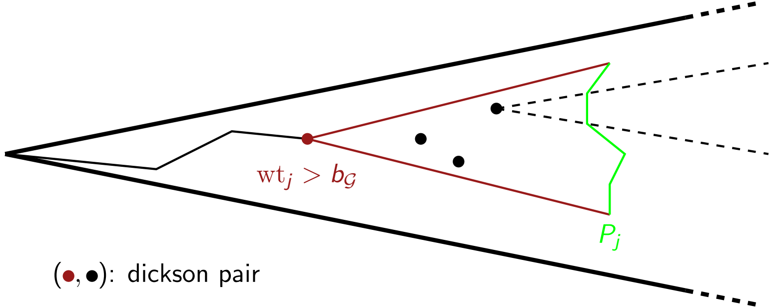

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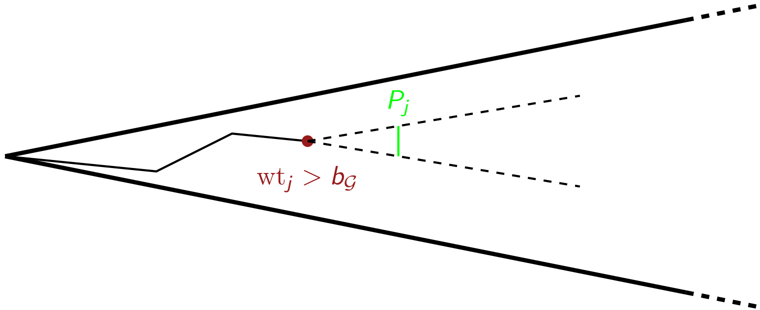

From RR Games to Mean-Payoff Games

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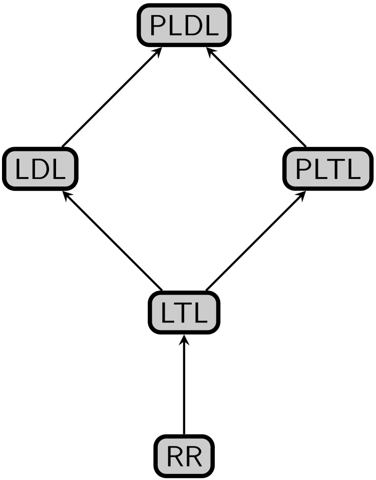
Theorem

Optimal strategy for mean-payoff game can be translated into optimal strategy for RR game.

Conclusion



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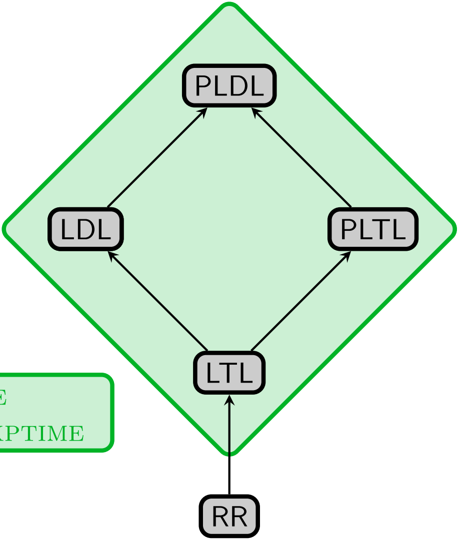

MC: PSPACE
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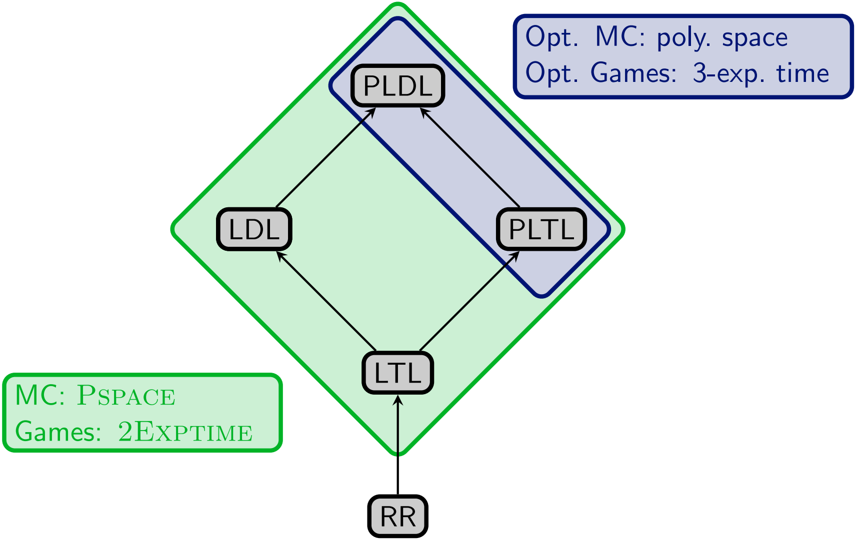
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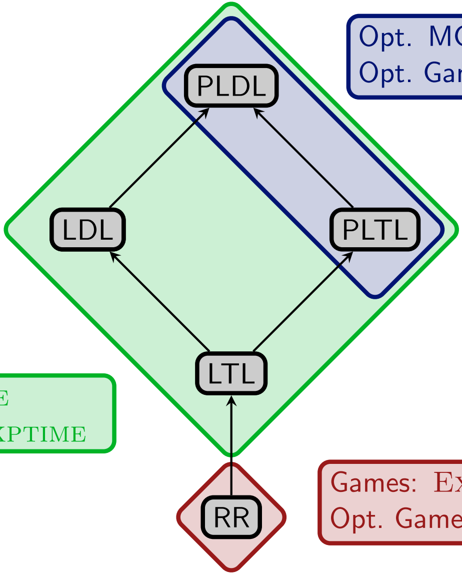

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