Playing Infinite Games in Finite Time

Joint work with John Fearnley,
Daniel Neider, and Roman Rabinovich

Martin Zimmermann

RWTH Aachen University

November 17th, 2011

Algosyn Workshop 2011, Rolduc

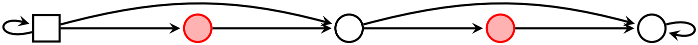
Introduction

How can you play infinite games in finite time? Aren't they infinite for a reason?

Introduction

How can you play infinite games in finite time? Aren't they infinite for a reason?

Introductory example

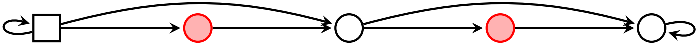


Player 0 wins play iff no red vertex is visited $\Rightarrow$ safety game.

Introduction

How can you play infinite games in finite time? Aren't they infinite for a reason?

Introductory example



Player 0 wins play iff no red vertex is visited $\Rightarrow$ safety game.

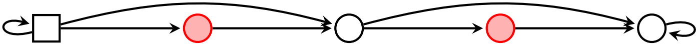
Observations:

- If a red vertex is reached, there is no point in continuing.

Introduction

How can you play infinite games in finite time? Aren't they infinite for a reason?

Introductory example



Player 0 wins play iff no red vertex is visited $\Rightarrow$ safety game.

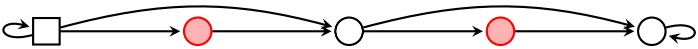
Observations:

- If a red vertex is reached, there is no point in continuing.
- Positional determinacy: stop a play when some vertex is visited for the second time.

Introduction

How can you play infinite games in finite time? Aren't they infinite for a reason?

Introductory example



Player 0 wins play iff no red vertex is visited ⇒ safety game.

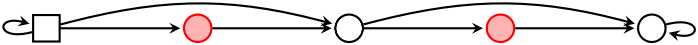
Observations:

- If a red vertex is reached, there is no point in continuing.
- Positional determinacy: stop a play when some vertex is visited for the second time.
- Generalizes to finite-state determinacy: stop a play when some memory state is repeated.

Introduction

How can you play infinite games in finite time? Aren't they infinite for a reason?

Introductory example



Player 0 wins play iff no red vertex is visited ⇒ safety game.

Observations:

- If a red vertex is reached, there is no point in continuing.
- Positional determinacy: stop a play when some vertex is visited for the second time.
- Generalizes to finite-state determinacy: stop a play when some memory state is repeated.

Can we play in finite time without relying on a memory structure?

Muller Games

Inspired by previous work of McNaughton on playing infinite games in finite time, we consider Muller games $(\mathcal{A}, \mathcal{F}_0, \mathcal{F}_1)$:

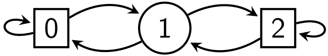
- arena $\mathcal{A}$ and partition $(\mathcal{F}_0, \mathcal{F}_1)$ containing the loops of $\mathcal{A}$.
- Player i wins ρ iff $\mathrm{Inf}(\rho) = \{v \mid \exists^\omega n \text{ s.t. } \rho_n = v\} \in \mathcal{F}_i$.

Muller Games

Inspired by previous work of McNaughton on playing infinite games in finite time, we consider Muller games $(\mathcal{A}, \mathcal{F}_0, \mathcal{F}_1)$:

- arena $\mathcal{A}$ and partition $(\mathcal{F}_0, \mathcal{F}_1)$ containing the loops of $\mathcal{A}$.
- Player i wins ρ iff $\mathrm{Inf}(\rho) = \{v \mid \exists^\omega n \text{ s.t. } \rho_n = v\} \in \mathcal{F}_i$.

Running example



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

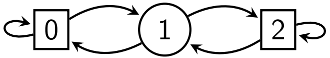
Player 0 has a winning strategy from every vertex: alternate between 0 and 2.

Muller Games

Inspired by previous work of McNaughton on playing infinite games in finite time, we consider Muller games $(\mathcal{A}, \mathcal{F}_0, \mathcal{F}_1)$:

- arena $\mathcal{A}$ and partition $(\mathcal{F}_0, \mathcal{F}_1)$ containing the loops of $\mathcal{A}$.
- Player i wins ρ iff $\mathrm{Inf}(\rho) = \{v \mid \exists^\omega n \text{ s.t. } \rho_n = v\} \in \mathcal{F}_i$.

Running example



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Player 0 has a winning strategy from every vertex: alternate between 0 and 2.

Remark

Muller games are not reducible to safety games.

Outline

1. **Playing Muller Games in Finite Time**
2. Solving Muller Games by Solving Safety Games
3. Conclusion

Scoring Functions

Let $F \subseteq V$, $F \neq \emptyset$.

Scoring Functions

Let $F \subseteq V$, $F \neq \emptyset$. For $v \in V$ define

$$\mathrm{Sc}_F(v) = \begin{cases} 1 & \text{if } F = \{v\}, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\mathrm{Acc}_F(v) = \begin{cases} \emptyset & \text{if } F = \{v\}, \\ F \cap \{v\} & \text{otherwise.} \end{cases}$$

Scoring Functions

Let $F \subseteq V$, $F \neq \emptyset$. For $v \in V$ and $w \in V^+$ define

$$\mathrm{Sc}_F(wv) = \begin{cases} 0 & \text{if } v \notin F, \\ \mathrm{Sc}_F(w) & \text{if } v \in F \wedge \mathrm{Acc}_F(w) \neq (F \setminus \{v\}), \\ \mathrm{Sc}_F(w) + 1 & \text{if } v \in F \wedge \mathrm{Acc}_F(w) = (F \setminus \{v\}), \end{cases}$$

and

$$\mathrm{Acc}_F(wv) = \begin{cases} \emptyset & \text{if } v \notin F, \\ \mathrm{Acc}_F(w) \cup \{v\} & \text{if } v \in F \wedge \mathrm{Acc}_F(w) \neq (F \setminus \{v\}), \\ \emptyset & \text{if } v \in F \wedge \mathrm{Acc}_F(w) = (F \setminus \{v\}). \end{cases}$$

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$								

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0							

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0						

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1					

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1				

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2			

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2		

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$								

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$							

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$						

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$					

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$				

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$			

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$		

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$								

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0							

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0						

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0					

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0				

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0			

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0		

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1
$\mathrm{Acc}_{\{0,1,2\}}$								

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$							

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$						

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$					

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$				

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$			

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$		

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\emptyset$

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices seen since last increase or reset of Sc_F.

Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\emptyset$

Remark

$F = \mathrm{Inf}(\rho) \Leftrightarrow \liminf_{n\to\infty} \mathrm{Sc}_F(\rho_0 \cdots \rho_n) = \infty$

Finite-time Muller Games

Two properties of scoring functions (informal versions):

1. If you play long enough (i.e., $k^{|V|}$ steps), some score value will be high (i.e., k).
2. At most one score value can increase at a time.

Finite-time Muller Games

Two properties of scoring functions (informal versions):

1. If you play long enough (i.e., $k^{|V|}$ steps), some score value will be high (i.e., k).
2. At most one score value can increase at a time.

Definition

Finite-time Muller game: $(\mathcal{A}, \mathcal{F}_0, \mathcal{F}_1, k)$ with threshold $k \geq 3$.

Finite-time Muller Games

Two properties of scoring functions (informal versions):

1. If you play long enough (i.e., $k^{|V|}$ steps), some score value will be high (i.e., k).
2. At most one score value can increase at a time.

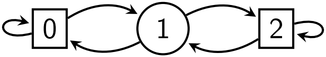
Definition

Finite-time Muller game: $(\mathcal{A}, \mathcal{F}_0, \mathcal{F}_1, k)$ with threshold $k \geq 3$.

Rules:

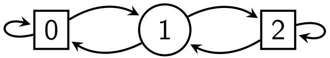
- Players move a token through the arena.
- Stop play w as soon as score of k is reached for the first time.
- There is a unique F such that $\mathrm{Sc}_F(w) = k$ (see above).
- Player i wins w iff $F \in \mathcal{F}_i$.

Two Examples



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

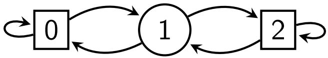
Two Examples



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

Two Examples

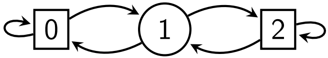


- $\mathcal{F}_0 = \{\{0, 1, 2\}, \{0\}, \{2\}\}$
- $\mathcal{F}_1 = \{\{0, 1\}, \{1, 2\}\}$

Losing player (Player 1) can enforce score of two:

1

Two Examples

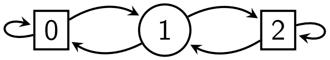


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \qquad \text{(w.l.o.g.)}$$

Two Examples

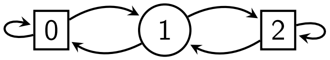


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \longrightarrow 2$$

Two Examples

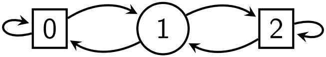


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

Two Examples



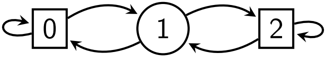
- $\mathcal{F}_0 = \{\{0, 1, 2\}, \{0\}, \{2\}\}$
- $\mathcal{F}_1 = \{\{0, 1\}, \{1, 2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

$\mathrm{Sc}_{\{1,2\}} = 2$

Two Examples

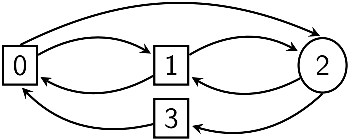


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

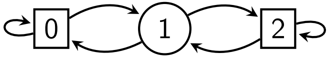
$$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

$\mathrm{Sc}_{\{1,2\}} = 2$



- $\mathcal{F}_0 = \{\{0,1\},\{1,2\},$
  $\{0,1,2,3\}\}$
- $\mathcal{F}_1 = \{\{0,1,2\},\{0,2,3\}\}$

Two Examples

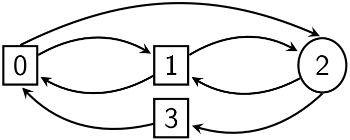


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

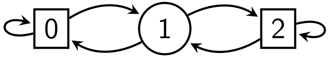
$\mathrm{Sc}_{\{1,2\}} = 2$



- $\mathcal{F}_0 = \{\{0,1\},\{1,2\},$
  $\{0,1,2,3\}\}$
- $\mathcal{F}_1 = \{\{0,1,2\},\{0,2,3\}\}$

Losing player (Player 1) is the first to reach a score of two:

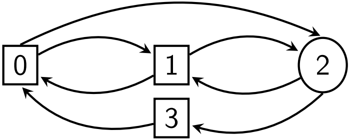
Two Examples



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$ $\mathrm{Sc}_{\{1,2\}} = 2$

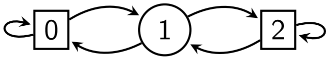


- $\mathcal{F}_0 = \{\{0,1\},\{1,2\},$
  $\{0,1,2,3\}\}$
- $\mathcal{F}_1 = \{\{0,1,2\},\{0,2,3\}\}$

Losing player (Player 1) is the first to reach a score of two:

3

Two Examples

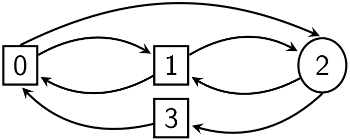


- $\mathcal{F}_0 = \{\{0, 1, 2\}, \{0\}, \{2\}\}$
- $\mathcal{F}_1 = \{\{0, 1\}, \{1, 2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

$\mathrm{Sc}_{\{1,2\}} = 2$

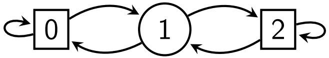


- $\mathcal{F}_0 = \{\{0, 1\}, \{1, 2\}, \{0, 1, 2, 3\}\}$
- $\mathcal{F}_1 = \{\{0, 1, 2\}, \{0, 2, 3\}\}$

Losing player (Player 1) is the first to reach a score of two:

$$3 \longrightarrow 0$$

Two Examples

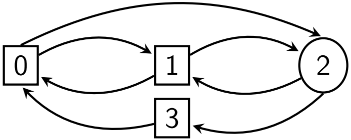


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

$\mathrm{Sc}_{\{1,2\}} = 2$

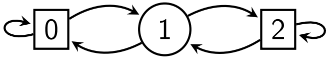


- $\mathcal{F}_0 = \{\{0,1\},\{1,2\},\{0,1,2,3\}\}$
- $\mathcal{F}_1 = \{\{0,1,2\},\{0,2,3\}\}$

Losing player (Player 1) is the first to reach a score of two:

$$3 \longrightarrow 0 \longrightarrow 2$$

Two Examples

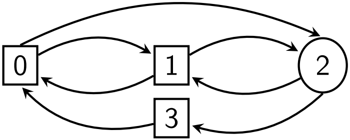


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

$\mathrm{Sc}_{\{1,2\}} = 2$

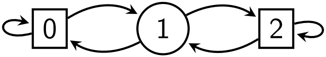


- $\mathcal{F}_0 = \{\{0,1\},\{1,2\},$
  $\{0,1,2,3\}\}$
- $\mathcal{F}_1 = \{\{0,1,2\},\{0,2,3\}\}$

Losing player (Player 1) is the first to reach a score of two:

$$3 \longrightarrow 0 \longrightarrow 2 \nearrow 1$$

Two Examples

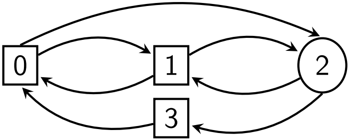


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

$\mathrm{Sc}_{\{1,2\}} = 2$

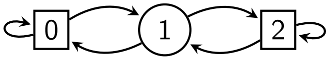


- $\mathcal{F}_0 = \{\{0,1\},\{1,2\},$
  $\{0,1,2,3\}\}$
- $\mathcal{F}_1 = \{\{0,1,2\},\{0,2,3\}\}$

Losing player (Player 1) is the first to reach a score of two:

$$3 \longrightarrow 0 \longrightarrow 2 \nearrow 1 \longrightarrow 0$$

Two Examples

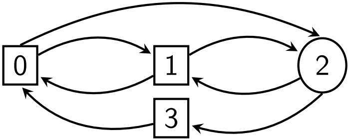


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

$\mathrm{Sc}_{\{1,2\}} = 2$

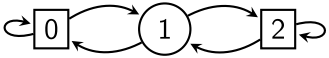


- $\mathcal{F}_0 = \{\{0,1\},\{1,2\},$
  $\{0,1,2,3\}\}$
- $\mathcal{F}_1 = \{\{0,1,2\},\{0,2,3\}\}$

Losing player (Player 1) is the first to reach a score of two:

$$3 \longrightarrow 0 \longrightarrow 2 \nearrow 1 \longrightarrow 0 \longrightarrow 1$$

Two Examples

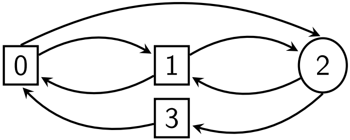


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

$\mathrm{Sc}_{\{1,2\}} = 2$

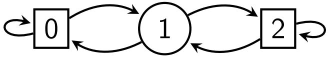


- $\mathcal{F}_0 = \{\{0,1\},\{1,2\},\{0,1,2,3\}\}$
- $\mathcal{F}_1 = \{\{0,1,2\},\{0,2,3\}\}$

Losing player (Player 1) is the first to reach a score of two:

$$3 \longrightarrow 0 \longrightarrow 2 \nearrow 1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2$$

Two Examples

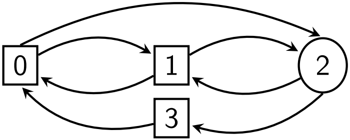


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

$\mathrm{Sc}_{\{1,2\}} = 2$



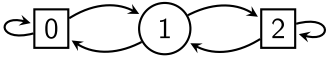
- $\mathcal{F}_0 = \{\{0,1\},\{1,2\},\{0,1,2,3\}\}$
- $\mathcal{F}_1 = \{\{0,1,2\},\{0,2,3\}\}$

Losing player (Player 1) is the first to reach a score of two:

$$3 \longrightarrow 0 \longrightarrow 2 \nearrow 1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2$$

$\mathrm{Sc}_{\{0,1,2\}} = 2$

Two Examples

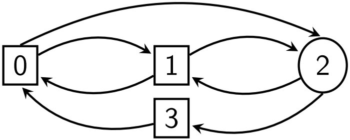


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

$\mathrm{Sc}_{\{1,2\}} = 2$



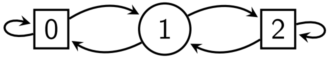
- $\mathcal{F}_0 = \{\{0,1\},\{1,2\},\{0,1,2,3\}\}$
- $\mathcal{F}_1 = \{\{0,1,2\},\{0,2,3\}\}$

Losing player (Player 1) is the first to reach a score of two:

$$3 \longrightarrow 0 \longrightarrow 2 \begin{cases} \nearrow 1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \\ \searrow 3 \end{cases}$$

$\mathrm{Sc}_{\{0,1,2\}} = 2$

Two Examples

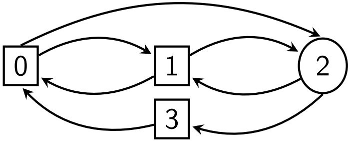


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

$\mathrm{Sc}_{\{1,2\}} = 2$



- $\mathcal{F}_0 = \{\{0,1\},\{1,2\},$
  $\{0,1,2,3\}\}$
- $\mathcal{F}_1 = \{\{0,1,2\},\{0,2,3\}\}$

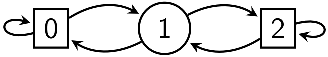
Losing player (Player 1) is the first to reach a score of two:

$$3 \longrightarrow 0 \longrightarrow 2 \nearrow 1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2$$

$$3 \longrightarrow 0 \longrightarrow 2 \searrow 3 \longrightarrow 0$$

$\mathrm{Sc}_{\{0,1,2\}} = 2$

Two Examples

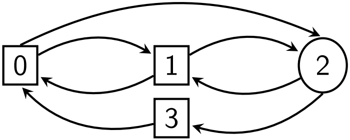


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

$\mathrm{Sc}_{\{1,2\}} = 2$



- $\mathcal{F}_0 = \{\{0,1\},\{1,2\},$
  $\{0,1,2,3\}\}$
- $\mathcal{F}_1 = \{\{0,1,2\},\{0,2,3\}\}$

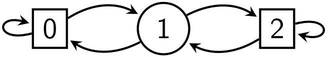
Losing player (Player 1) is the first to reach a score of two:

$$3 \longrightarrow 0 \longrightarrow 2 \nearrow 1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2$$

$\mathrm{Sc}_{\{0,1,2\}} = 2$

$$3 \longrightarrow 0 \longrightarrow 2 \searrow 3 \longrightarrow 0 \longrightarrow 2$$

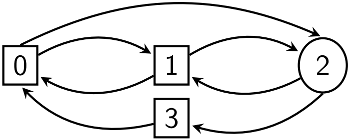
Two Examples



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Losing player (Player 1) can enforce score of two:

$$1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$ $\mathrm{Sc}_{\{1,2\}} = 2$



- $\mathcal{F}_0 = \{\{0,1\},\{1,2\},$
  $\{0,1,2,3\}\}$
- $\mathcal{F}_1 = \{\{0,1,2\},\{0,2,3\}\}$

Losing player (Player 1) is the first to reach a score of two:

$$3 \longrightarrow 0 \longrightarrow 2 \nearrow 1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2$$ $\mathrm{Sc}_{\{0,1,2\}} = 2$

$$3 \longrightarrow 0 \longrightarrow 2 \searrow 3 \longrightarrow 0 \longrightarrow 2$$ $\mathrm{Sc}_{\{0,2,3\}} = 2$

Results

Theorem (FZ10)

The winning regions in a Muller game $(\mathcal{A}, \mathcal{F}_0, \mathcal{F}_1)$ *and in the finite-time Muller game* $(\mathcal{A}, \mathcal{F}_0, \mathcal{F}_1, 3)$ *coincide.*

Results

Theorem (FZ10)

The winning regions in a Muller game $(\mathcal{A}, \mathcal{F}_0, \mathcal{F}_1)$ and in the finite-time Muller game $(\mathcal{A}, \mathcal{F}_0, \mathcal{F}_1, 3)$ coincide.

Stronger statement, which implies the theorem:

Lemma (FZ10)

On her winning region, Player i can prevent her opponent from ever reaching a score of 3 for every set $F \in \mathcal{F}_{1-i}$.

Results

Theorem (FZ10)

The winning regions in a Muller game $(\mathcal{A}, \mathcal{F}_0, \mathcal{F}_1)$ and in the finite-time Muller game $(\mathcal{A}, \mathcal{F}_0, \mathcal{F}_1, 3)$ coincide.

Stronger statement, which implies the theorem:

Lemma (FZ10)

On her winning region, Player i can prevent her opponent from ever reaching a score of 3 for every set $F \in \mathcal{F}_{1-i}$.

Corollary

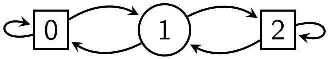
Two "reductions": Muller game to ..

1. *.. reachability game on unravelling up to score 3.*
2. *.. safety game: see next slides.*

Outline

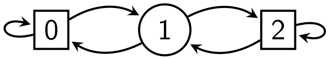
1. Playing Muller Games in Finite Time
2. **Solving Muller Games by Solving Safety Games**
3. Conclusion

“Reducing“ Muller games to Safety Games



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

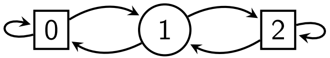
“Reducing“ Muller games to Safety Games



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Idea: track of Player 1's scores and avoid $\mathrm{Sc}_F = 3$ for $F \in \mathcal{F}_1$.

“Reducing“ Muller games to Safety Games



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

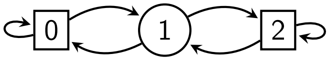
Idea: track of Player 1's scores and avoid $\mathrm{Sc}_F = 3$ for $F \in \mathcal{F}_1$.

- Ignore scores of Player 0.
- Identify plays having the same scores and accumulators for Player 1: $w =_{\mathcal{F}_1} w'$ iff $\mathrm{last}(w) = \mathrm{last}(w')$ and for all $F \in \mathcal{F}_1$:

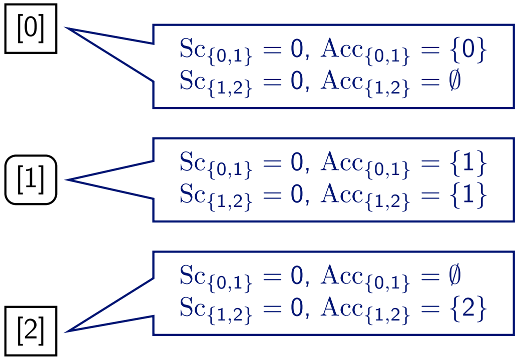
$$\mathrm{Sc}_F(w) = \mathrm{Sc}_F(w') \text{ and } \mathrm{Acc}_F(w) = \mathrm{Acc}(w')$$

- Build $=_{\mathcal{F}_1}$-quotient of unravelling up to score 3 for Player 1.
- Winning condition for Player 0: avoid $\mathrm{Sc}_F = 3$ for all $F \in \mathcal{F}_1$.

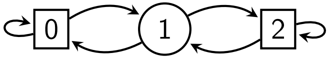
“Reducing“ Muller games to Safety Games



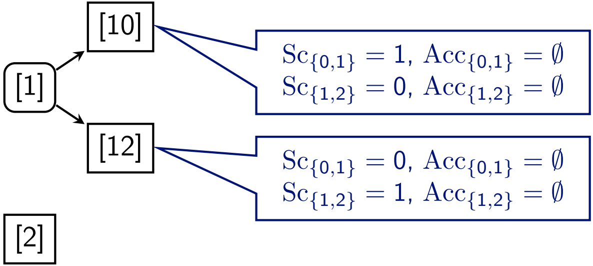
- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$



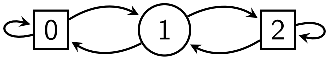
“Reducing“ Muller games to Safety Games



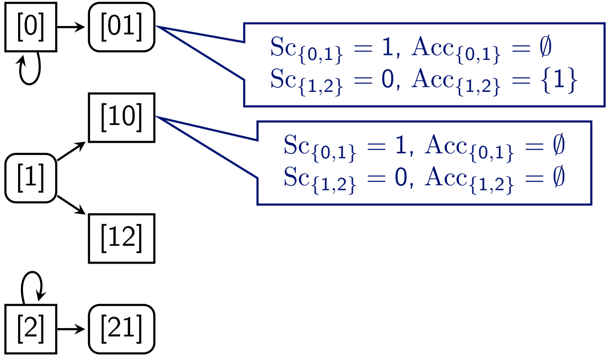
- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$



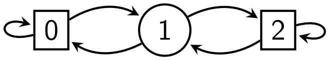
"Reducing" Muller games to Safety Games



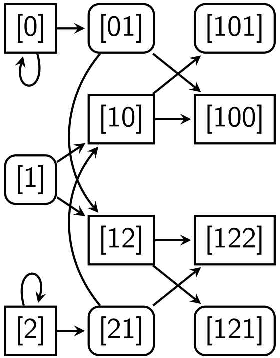
- $\mathcal{F}_0 = \{\{0, 1, 2\}, \{0\}, \{2\}\}$
- $\mathcal{F}_1 = \{\{0, 1\}, \{1, 2\}\}$



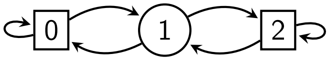
"Reducing" Muller games to Safety Games



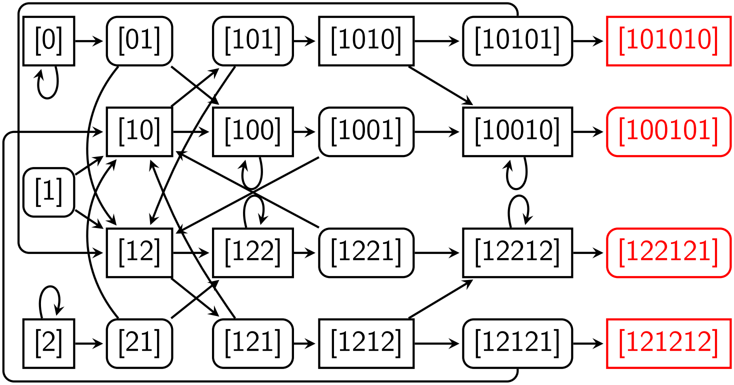
- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$



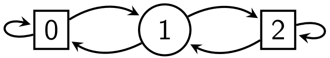
“Reducing“ Muller games to Safety Games



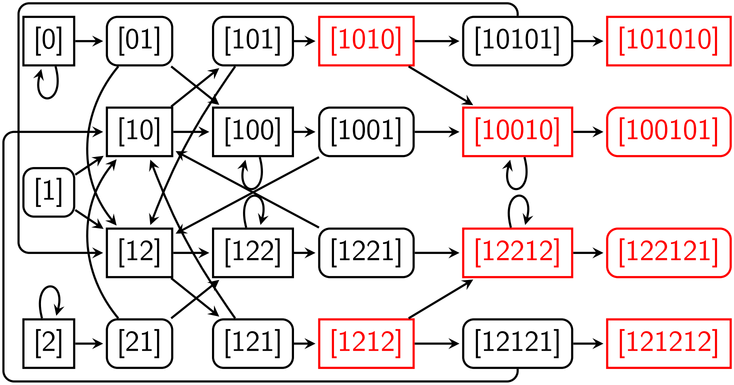
- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$



“Reducing“ Muller games to Safety Games



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$



Results

Theorem (NRZ11)

1. *Player i wins the Muller game from v iff she wins the safety game from $[v]_{=_{\mathcal{F}_1}}$.*
2. *Player 0's winning region in the safety game can be turned into finite-state winning strategy for her in the Muller game.*
3. *Size of the safety game $(n!)^3$.*

Results

Theorem (NRZ11)

1. *Player i wins the Muller game from v iff she wins the safety game from $[v]_{=_{\mathcal{F}_1}}$.*
2. *Player 0's winning region in the safety game can be turned into finite-state winning strategy for her in the Muller game.*
3. *Size of the safety game $(n!)^3$.*

Remarks:

- Size of parity game in LAR-reduction $n!$. But: safety games allow much simpler algorithms.

Results

Theorem (NRZ11)

1. *Player i wins the Muller game from v iff she wins the safety game from $[v]_{=_{\mathcal{F}_1}}$.*
2. *Player 0's winning region in the safety game can be turned into finite-state winning strategy for her in the Muller game.*
3. *Size of the safety game $(n!)^3$.*

Remarks:

- Size of parity game in LAR-reduction $n!$. But: safety games allow much simpler algorithms.
- 2. does not hold for Player 1.

Results

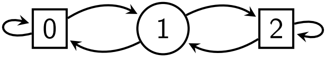
Theorem (NRZ11)

1. *Player i wins the Muller game from v iff she wins the safety game from $[v]_{=_{\mathcal{F}_1}}$.*
2. *Player 0's winning region in the safety game can be turned into finite-state winning strategy for her in the Muller game.*
3. *Size of the safety game $(n!)^3$.*

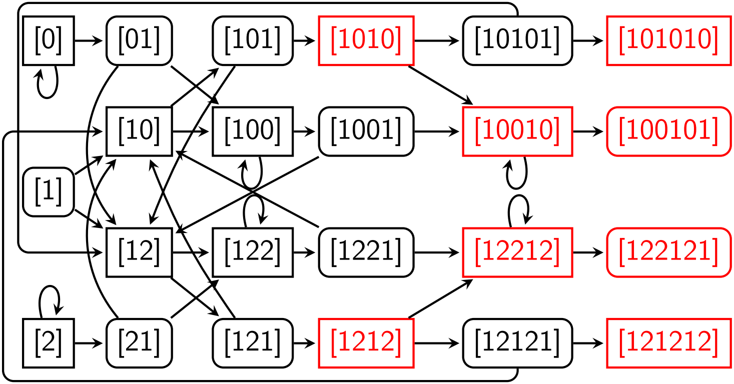
Remarks:

- Size of parity game in LAR-reduction $n!$. But: safety games allow much simpler algorithms.
- 2. does not hold for Player 1.
- Not a reduction in the classical sense: not every play of the Muller game can be mapped to a play in the safety game.

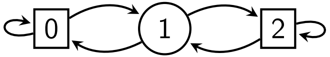
Proof Idea: Safety to Muller



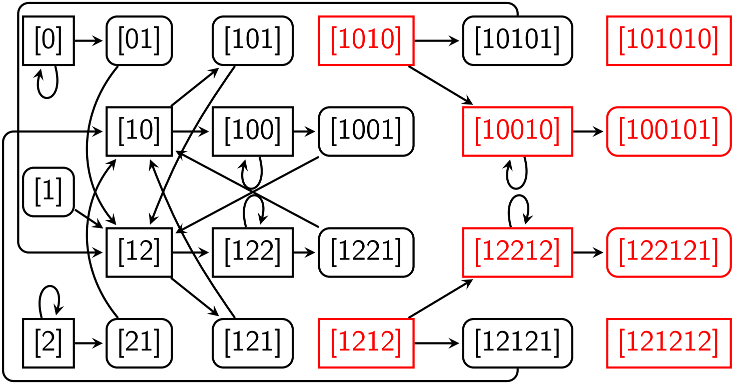
- $\mathcal{F}_0 = \{\{0, 1, 2\}, \{0\}, \{2\}\}$
- $\mathcal{F}_1 = \{\{0, 1\}, \{1, 2\}\}$



Proof Idea: Safety to Muller

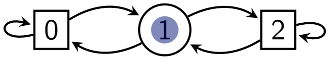


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

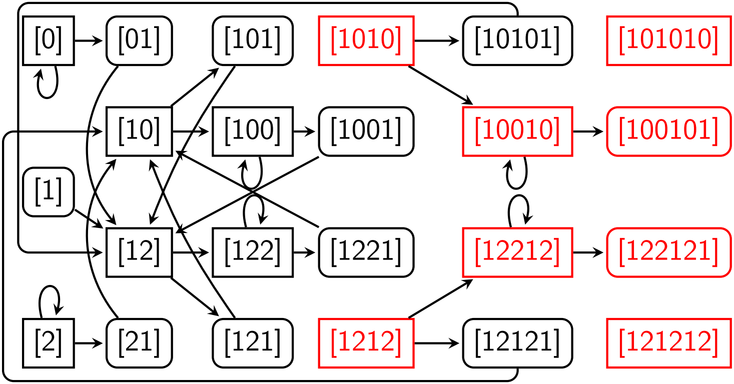


Pick a winning strategy for the safety game..

Proof Idea: Safety to Muller

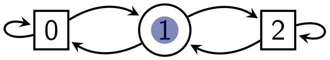


- $\mathcal{F}_0 = \{\{0, 1, 2\}, \{0\}, \{2\}\}$
- $\mathcal{F}_1 = \{\{0, 1\}, \{1, 2\}\}$

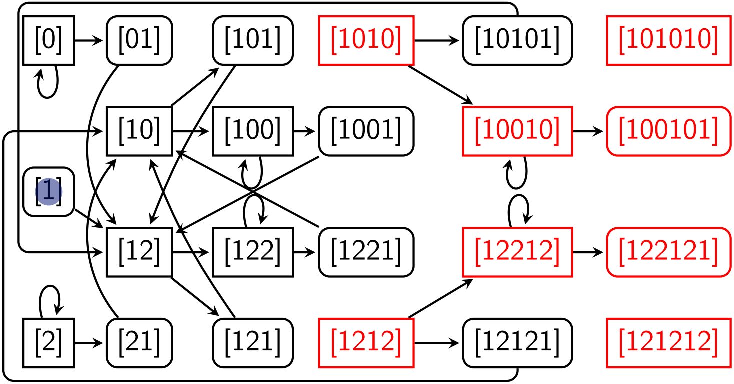


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

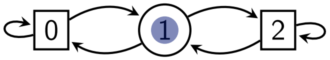


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

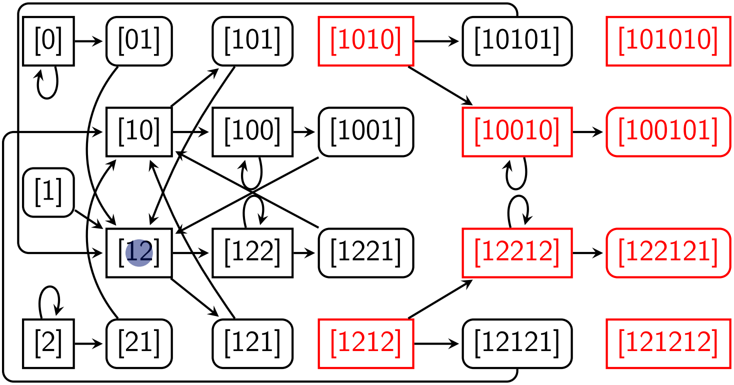


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

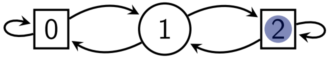


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

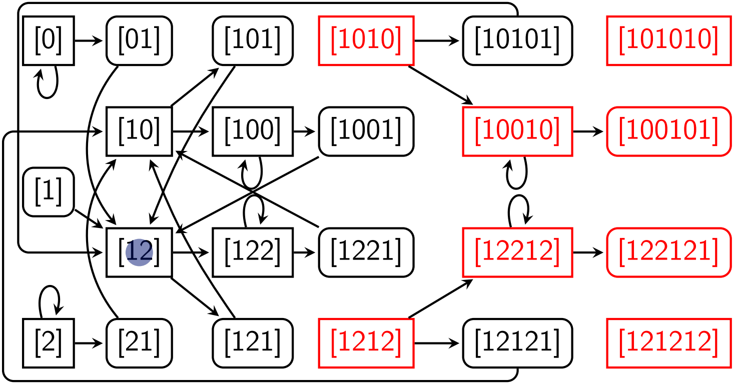


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

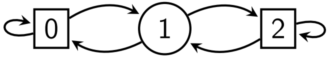


- $\mathcal{F}_0 = \{\{0, 1, 2\}, \{0\}, \{2\}\}$
- $\mathcal{F}_1 = \{\{0, 1\}, \{1, 2\}\}$

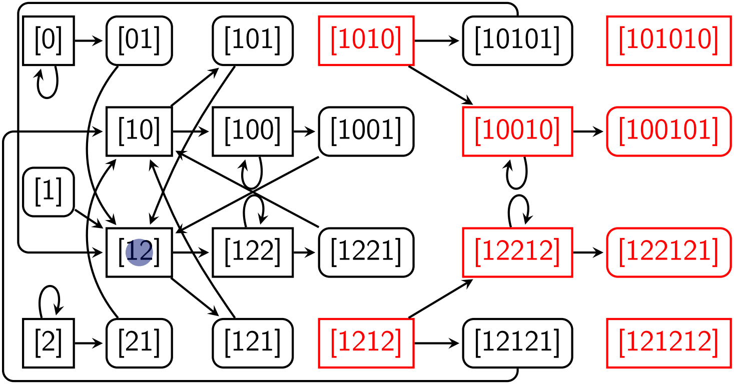


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

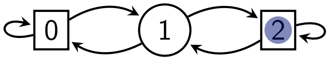


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

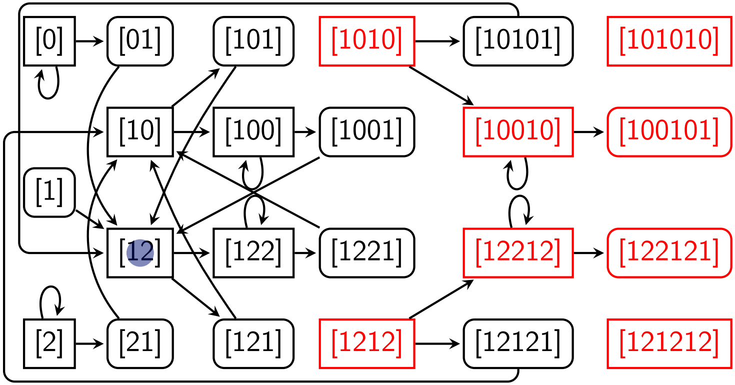


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

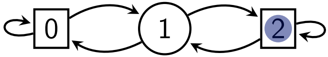


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

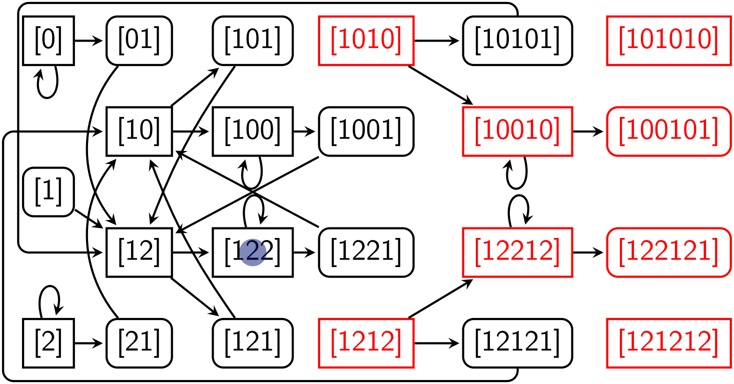


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

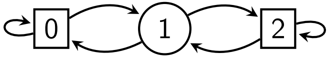


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

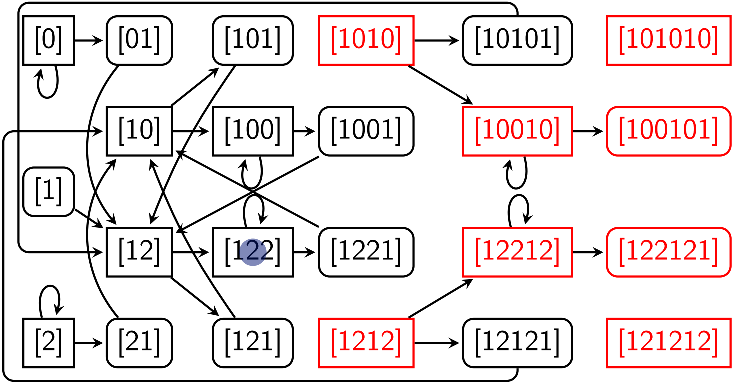


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

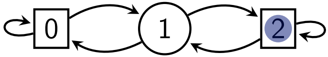


- $\mathcal{F}_0 = \{\{0, 1, 2\}, \{0\}, \{2\}\}$
- $\mathcal{F}_1 = \{\{0, 1\}, \{1, 2\}\}$

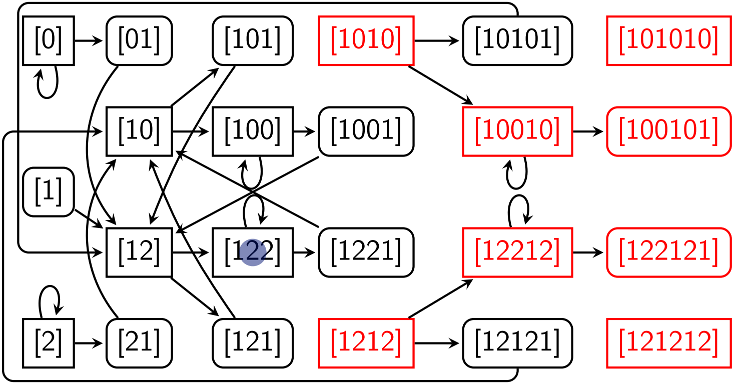


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

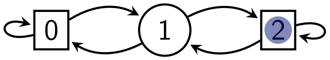


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

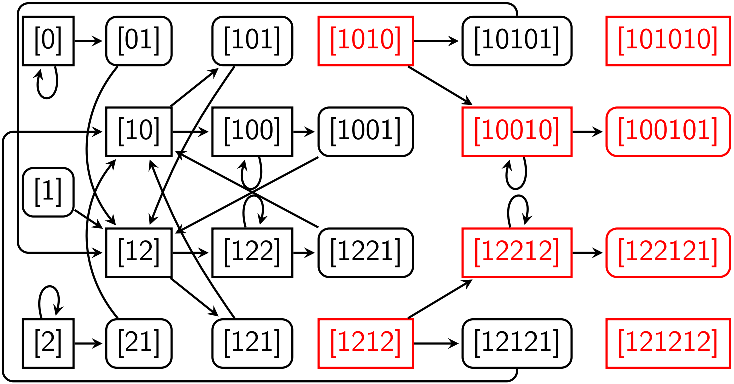


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

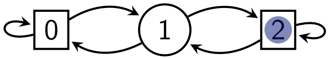


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

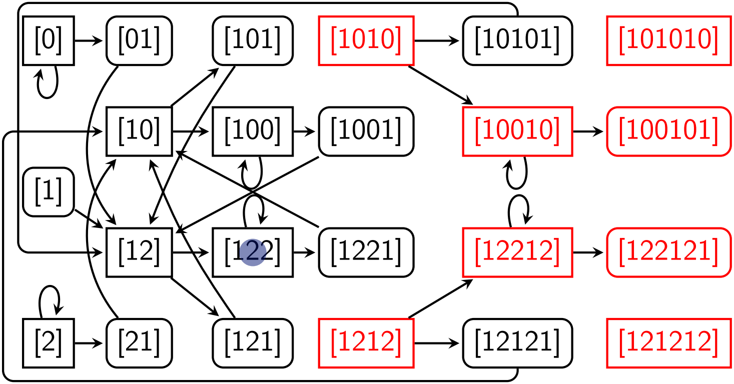


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

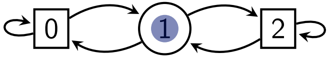


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

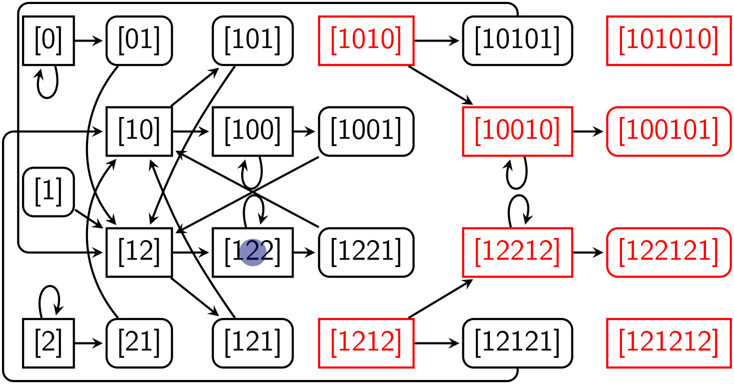


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

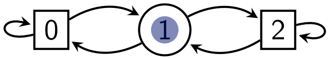


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

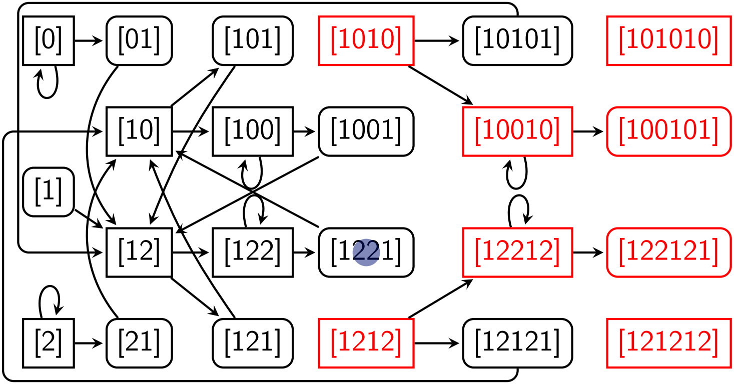


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

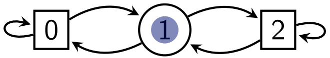


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

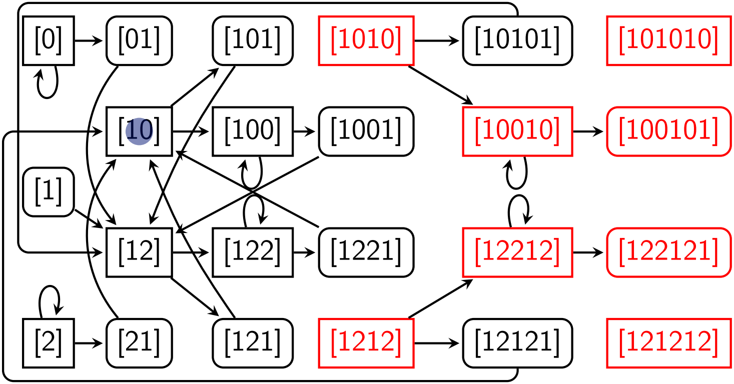


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

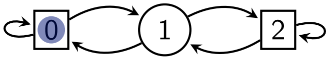


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

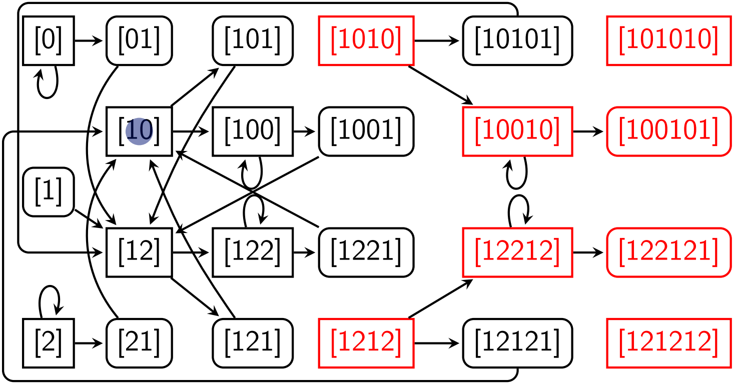


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

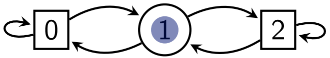


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

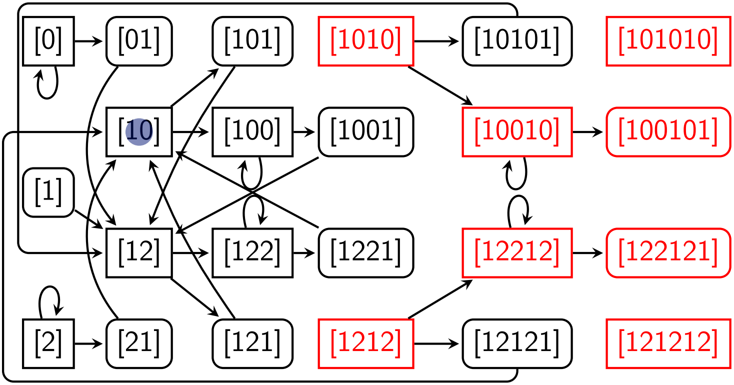


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

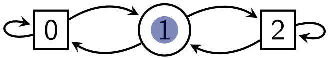


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

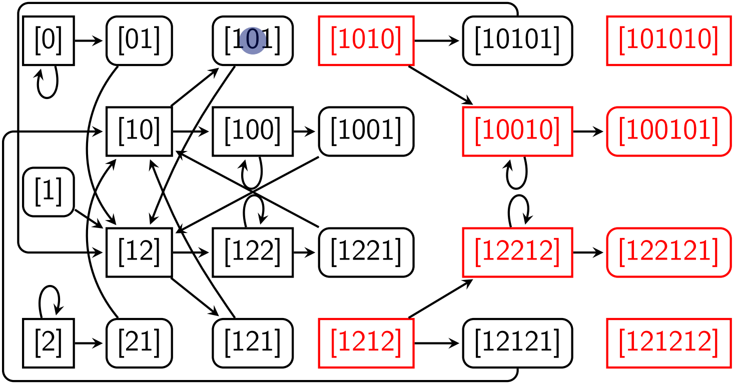


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

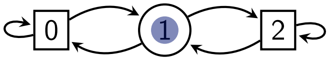


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

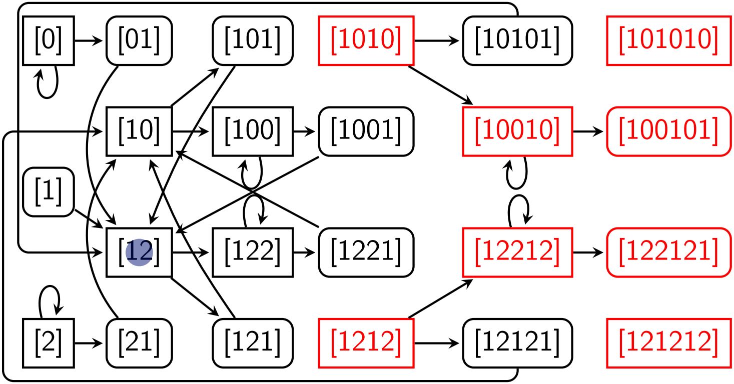


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

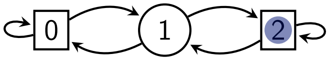


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

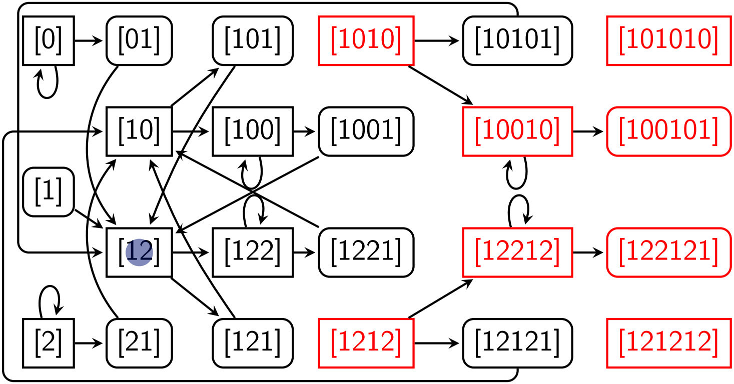


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

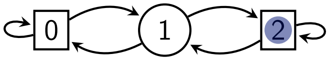


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

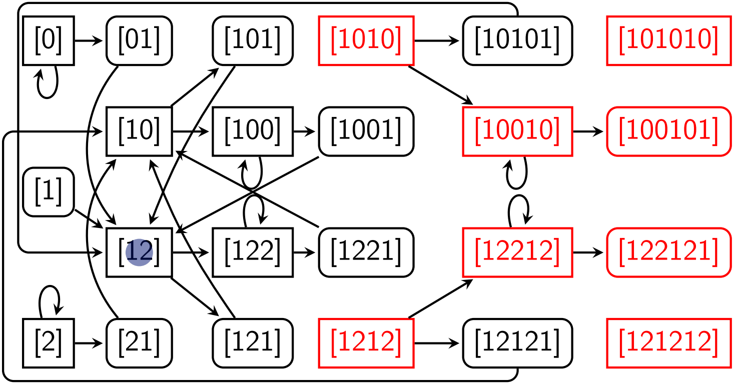


Pick a winning strategy for the safety game.. and let's play.

Proof Idea: Safety to Muller

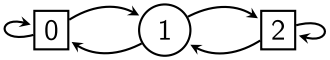


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

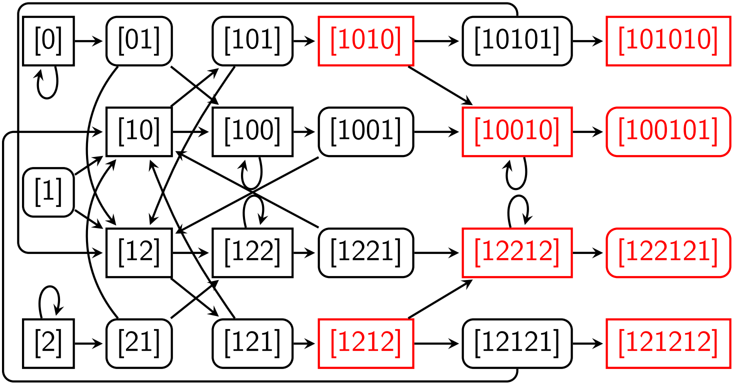


Sc_F for $F \in F_1$ in Muller game bounded by 2 $\Rightarrow$ winning strategy

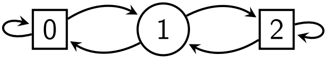
Proof Idea: Muller to Safety



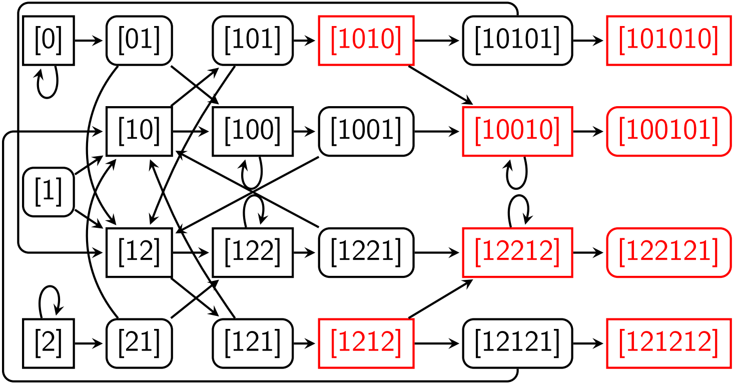
- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$



Proof Idea: Muller to Safety

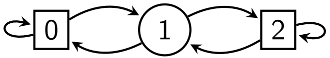


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

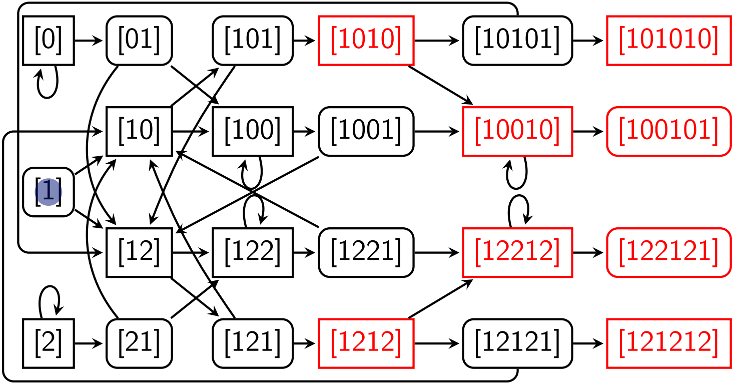


Pick a winning strategy for the Muller game that bounds Player 1's scores by two..

Proof Idea: Muller to Safety

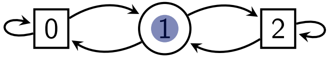


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

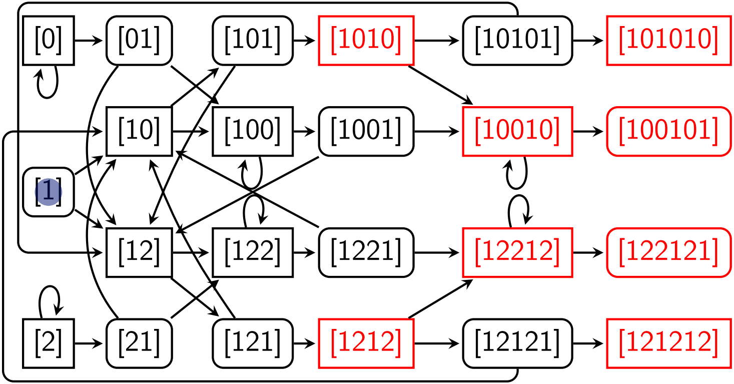


Pick a winning strategy for the Muller game that bounds Player 1's scores by two.. and let's play.

Proof Idea: Muller to Safety

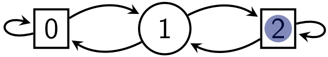


- $\mathcal{F}_0 = \{\{0, 1, 2\}, \{0\}, \{2\}\}$
- $\mathcal{F}_1 = \{\{0, 1\}, \{1, 2\}\}$

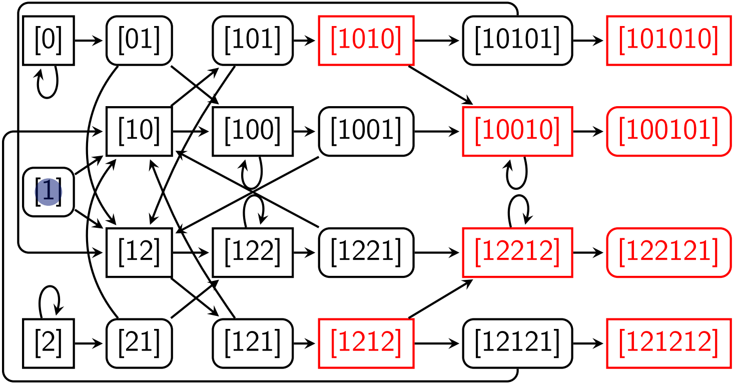


Pick a winning strategy for the Muller game that bounds Player 1's scores by two.. and let's play.

Proof Idea: Muller to Safety

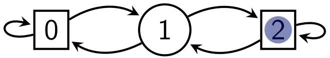


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

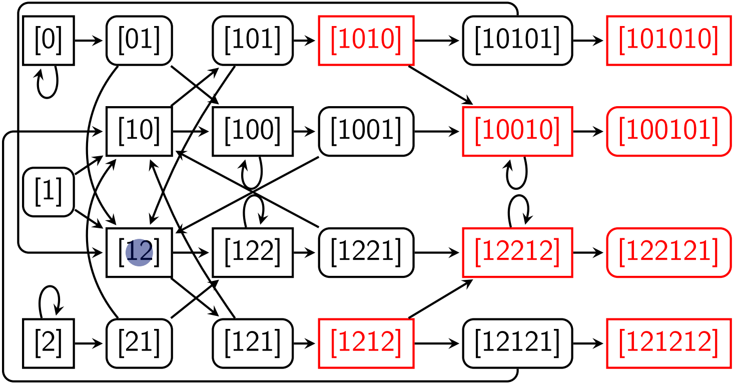


Pick a winning strategy for the Muller game that bounds Player 1's scores by two.. and let's play.

Proof Idea: Muller to Safety

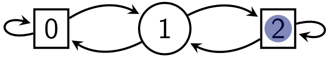


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

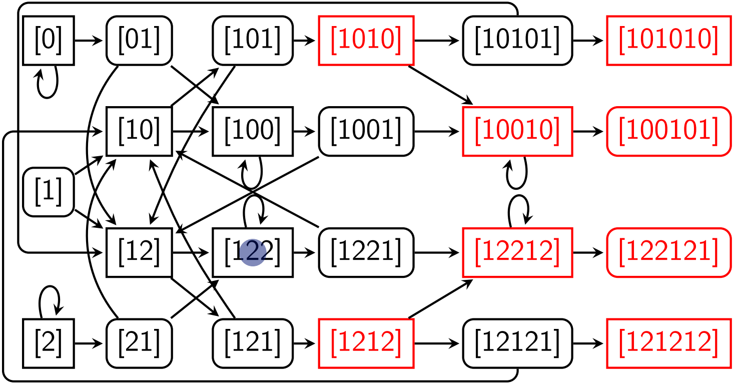


Pick a winning strategy for the Muller game that bounds Player 1's scores by two.. and let's play.

Proof Idea: Muller to Safety

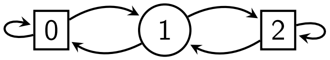


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

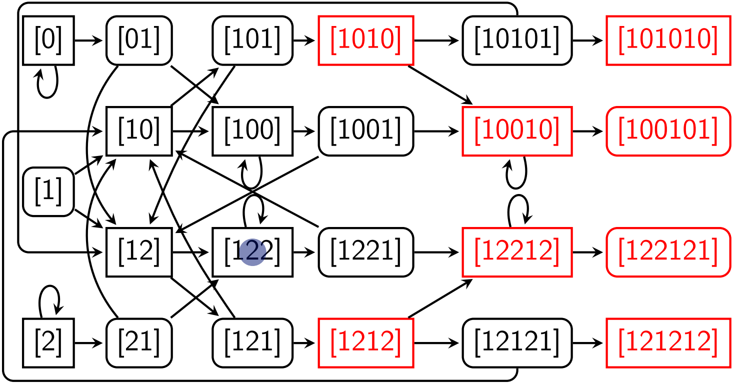


Pick a winning strategy for the Muller game that bounds Player 1's scores by two.. and let's play.

Proof Idea: Muller to Safety

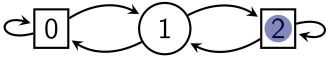


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

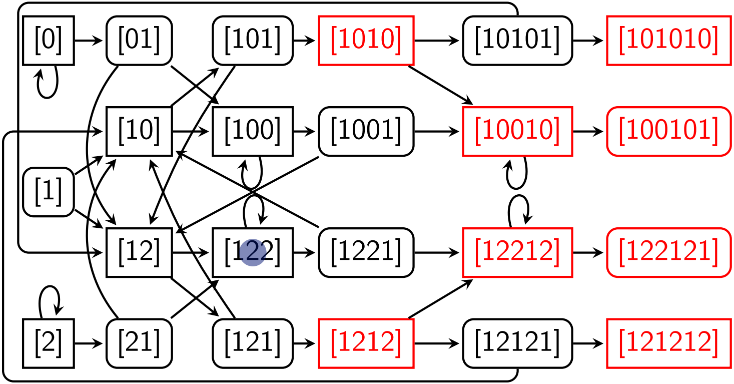


Pick a winning strategy for the Muller game that bounds Player 1's scores by two.. and let's play.

Proof Idea: Muller to Safety

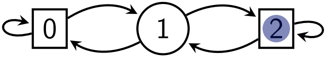


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

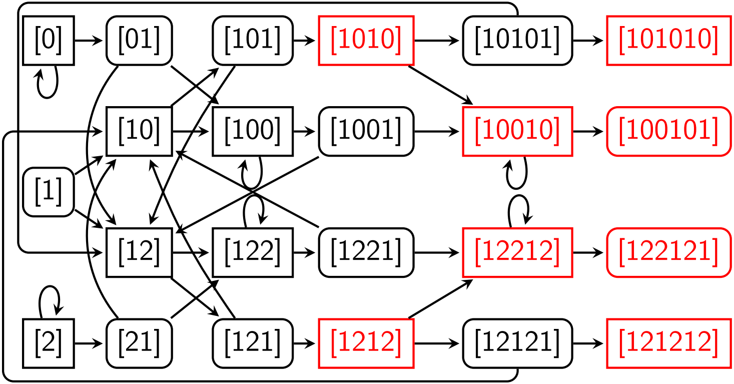


Pick a winning strategy for the Muller game that bounds Player 1's scores by two.. and let's play.

Proof Idea: Muller to Safety

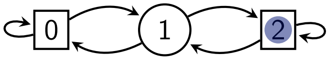


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

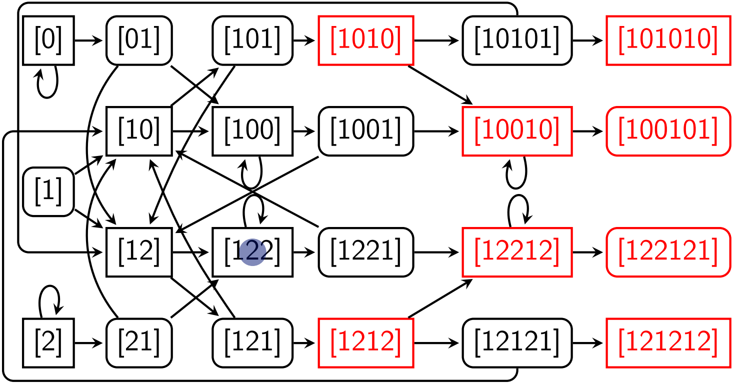


Pick a winning strategy for the Muller game that bounds Player 1's scores by two.. and let's play.

Proof Idea: Muller to Safety

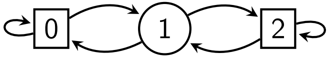


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

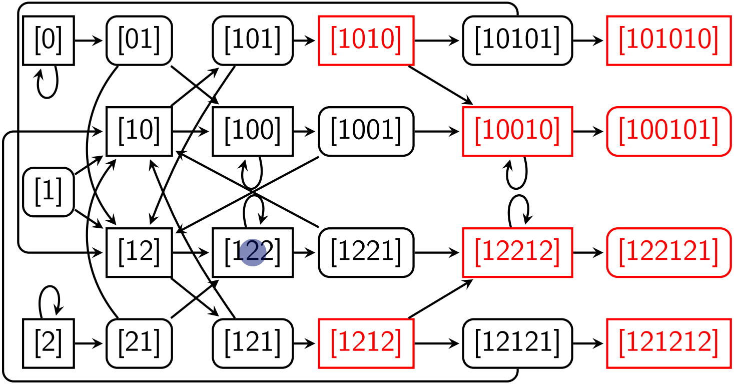


Pick a winning strategy for the Muller game that bounds Player 1's scores by two.. and let's play.

Proof Idea: Muller to Safety

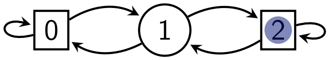


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

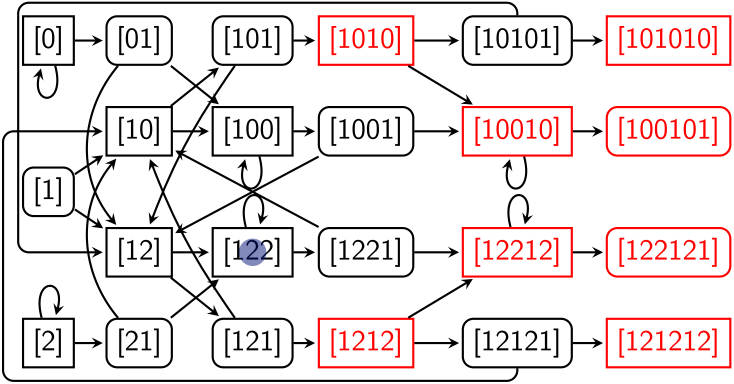


Pick a winning strategy for the Muller game that bounds Player 1's scores by two.. and let's play.

Proof Idea: Muller to Safety

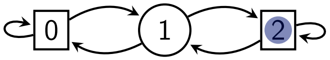


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

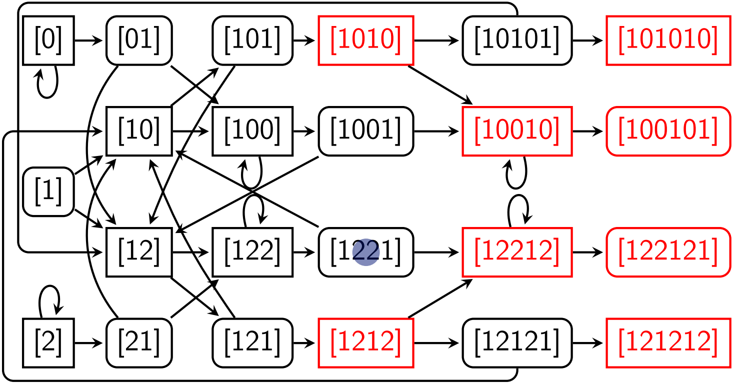


Pick a winning strategy for the Muller game that bounds Player 1's scores by two.. and let's play.

Proof Idea: Muller to Safety

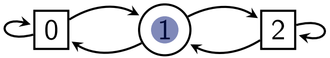


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

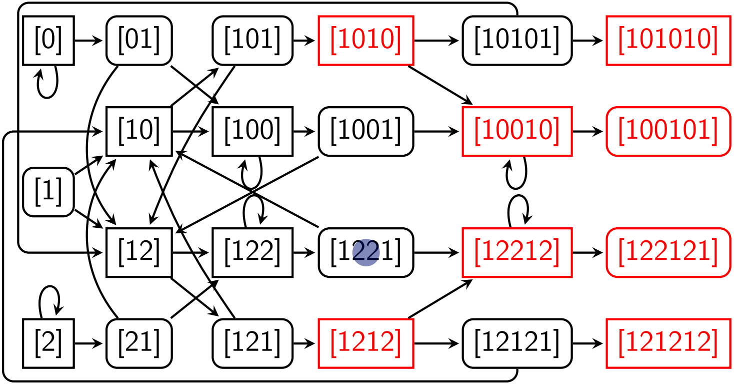


Pick a winning strategy for the Muller game that bounds Player 1's scores by two.. and let's play.

Proof Idea: Muller to Safety

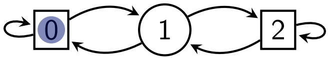


- $\mathcal{F}_0 = \{\{0, 1, 2\}, \{0\}, \{2\}\}$
- $\mathcal{F}_1 = \{\{0, 1\}, \{1, 2\}\}$

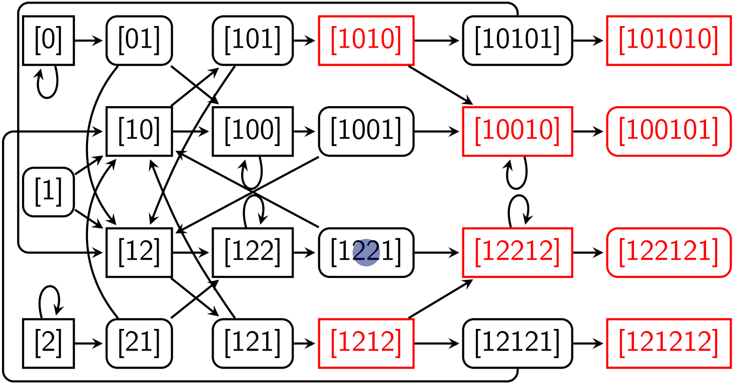


Pick a winning strategy for the Muller game that bounds Player 1's scores by two.. and let's play.

Proof Idea: Muller to Safety

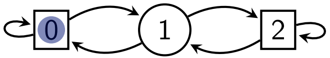


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

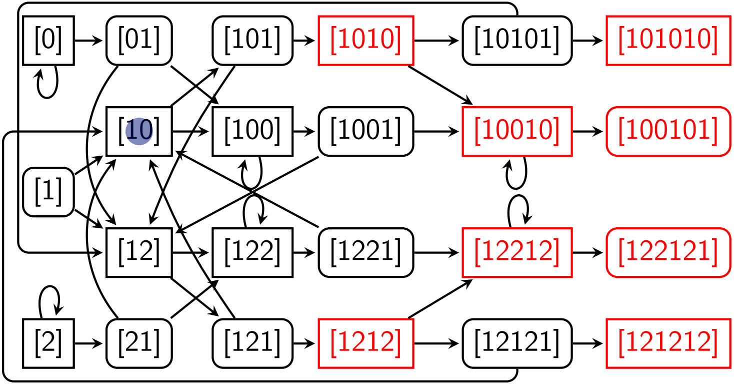


Pick a winning strategy for the Muller game that bounds Player 1's scores by two.. and let's play.

Proof Idea: Muller to Safety



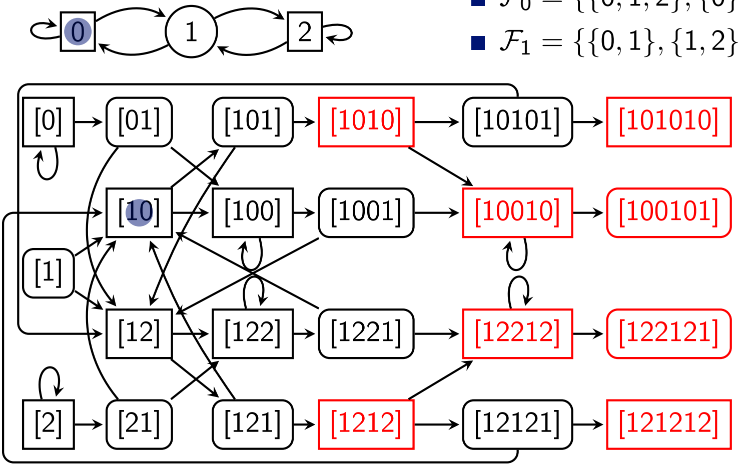
- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$



Pick a winning strategy for the Muller game that bounds Player 1's scores by two.. and let's play.

Proof Idea: Muller to Safety

- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$



Score of three is avoidable from every prefix ⇒ red vertices never reached ⇒ winning strategy.

Outline

1. Playing Muller Games in Finite Time

2. Solving Muller Games by Solving Safety Games

3. **Conclusion**

Conclusion

You can play Muller games in finite time!

- New algorithm for Muller games: just solve the safety game.
- New memory structure for Muller games: maximal elements of winning region suffice (antichain).
- New concept: permissive strategies for Muller games.
- Same constructions applicable for many other types of games.

Conclusion

You can play Muller games in finite time!

- New algorithm for Muller games: just solve the safety game.
- New memory structure for Muller games: maximal elements of winning region suffice (antichain).
- New concept: permissive strategies for Muller games.
- Same constructions applicable for many other types of games.

Ongoing and future work:

- Progress measure algorithm for Muller games?
- Is there a tradeoff between size and quality of a strategy?
- Can you play infinite games in infinite arenas in finite time?