Reducing ω-regular Specifications to Safety Conditions

Joint work with
John Fearnley (University of Liverpool)
Daniel Neider (RWTH Aachen University)
Roman Rabinovich (TU Berlin)

Martin Zimmermann

Saarland University

March 5th, 2014

AVACS Meeting, Oldenburg, Germany

ω-regular Specifications

- ω-regular expressions

ω-regular Specifications

- ω-regular expressions
- Monadic second-order logic with one successor

ω-regular Specifications

- ω-regular expressions
- Monadic second-order logic with one successor
- Non-deterministic automata with Büchi acceptance
  $(Q, \Sigma, q_0, \Delta, F)$ with $F \subseteq Q$ and

  $$q_0 q_1 q_2 \cdots \text{ accepting } \Leftrightarrow \mathrm{Inf}(q_0 q_1 q_2 \cdots) \cap F \neq \emptyset$$

ω-regular Specifications

- ω-regular expressions
- Monadic second-order logic with one successor
- Non-deterministic automata with Büchi acceptance
- Deterministic automata with Muller acceptance
  $(Q, \Sigma, q_0, \delta, \mathcal{F})$ with $\mathcal{F} \subseteq 2^Q$ and
  $$q_0 q_1 q_2 \cdots \text{ accepting } \Leftrightarrow \mathrm{Inf}(q_0 q_1 q_2 \cdots) \in \mathcal{F}$$

ω-regular Specifications

- ω-regular expressions
- Monadic second-order logic with one successor
- Non-deterministic automata with Büchi acceptance
- Deterministic automata with Muller acceptance
- Many other acceptance conditions: parity, Rabin, Streett, ..

ω-regular Specifications

- ω-regular expressions
- Monadic second-order logic with one successor
- Non-deterministic automata with Büchi acceptance
- Deterministic automata with Muller acceptance
- Many other acceptance conditions: parity, Rabin, Streett, ..

Generality: Every acceptance condition that only depends on the states visited infinitely often is a Muller condition.

ω-regular Specifications

- ω-regular expressions
- Monadic second-order logic with one successor
- Non-deterministic automata with Büchi acceptance
- Deterministic automata with Muller acceptance
- Many other acceptance conditions: parity, Rabin, Streett, ..

Generality: Every acceptance condition that only depends on the states visited infinitely often is a Muller condition.

- Non-deterministic automata with safety acceptance

ω-regular Specifications

- ω-regular expressions
- Monadic second-order logic with one successor
- Non-deterministic automata with Büchi acceptance
- Deterministic automata with Muller acceptance
- Many other acceptance conditions: parity, Rabin, Streett, ..

Generality: Every acceptance condition that only depends on the states visited infinitely often is a Muller condition.

- Non-deterministic automata with safety acceptance
  $(Q, \Sigma, q_0, \Delta, F)$ with $F \subseteq Q$ and

  $$q_0 q_1 q_2 \cdots \text{ accepting } \Leftrightarrow \mathrm{Occ}(q_0 q_1 q_2 \cdots) \subseteq F$$

ω-regular Specifications

- ω-regular expressions
- Monadic second-order logic with one successor
- Non-deterministic automata with Büchi acceptance
- Deterministic automata with Muller acceptance
- Many other acceptance conditions: parity, Rabin, Streett, ..

Generality: Every acceptance condition that only depends on the states visited infinitely often is a Muller condition.

- Non-deterministic automata with safety acceptance

Weaker: not every ω-regular language is a safety condition.

ω-regular Specifications

- ω-regular expressions
- Monadic second-order logic with one successor
- Non-deterministic automata with Büchi acceptance
- Deterministic automata with Muller acceptance
- Many other acceptance conditions: parity, Rabin, Streett, ..

Generality: Every acceptance condition that only depends on the states visited infinitely often is a Muller condition.

- Non-deterministic automata with safety acceptance

Weaker: not every ω-regular language is a safety condition.
Is it nevertheless possible to turn every Muller condition into an *equivalent* safety condition? (under which equivalence?)

Upside: simpler algorithms for safety conditions

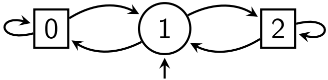
Muller Games

We study this question in a more general setting: infinite games.

Muller Games

We study this question in a more general setting: infinite games.

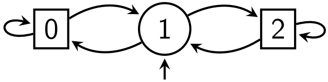
Running example



Muller Games

We study this question in a more general setting: infinite games.

Running example

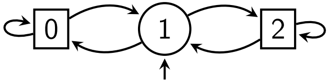


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Muller Games

We study this question in a more general setting: infinite games.

Running example



- $\mathcal{F}_0 = \{\{0, 1, 2\}, \{0\}, \{2\}\}$
- $\mathcal{F}_1 = \{\{0, 1\}, \{1, 2\}\}$

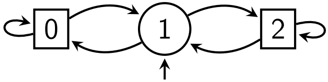
Formally: Muller game $(\mathcal{A}, \mathcal{F}_0, \mathcal{F}_1)$ with

- Arena $\mathcal{A} = (V, V_0, V_1, E, v)$ and partition $(\mathcal{F}_0, \mathcal{F}_1)$ of 2^V.
- Player i wins play ρ iff $\mathrm{Inf}(\rho) \in \mathcal{F}_i$.

Muller Games

We study this question in a more general setting: infinite games.

Running example



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Formally: Muller game $(\mathcal{A}, \mathcal{F}_0, \mathcal{F}_1)$ with

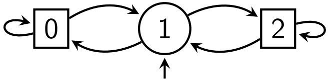
- Arena $\mathcal{A} = (V, V_0, V_1, E, v)$ and partition $(\mathcal{F}_0, \mathcal{F}_1)$ of 2^V.
- Player i wins play ρ iff $\mathrm{Inf}(\rho) \in \mathcal{F}_i$.

Emptiness of (non-deterministic) Muller automata and universality of deterministic Muller automata are one-player Muller games

Muller Games

We study this question in a more general setting: infinite games.

Running example



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Formally: Muller game $(\mathcal{A}, \mathcal{F}_0, \mathcal{F}_1)$ with

- Arena $\mathcal{A} = (V, V_0, V_1, E, v)$ and partition $(\mathcal{F}_0, \mathcal{F}_1)$ of 2^V.
- Player i wins play ρ iff $\mathrm{Inf}(\rho) \in \mathcal{F}_i$.

Emptiness of (non-deterministic) Muller automata and universality of deterministic Muller automata are one-player Muller games

Our goal: give winner-preserving reduction from Muller to safety games.

Playing Muller Games in Finite Time

Robert McNaughton:

> *We believe that infinite games might have an interest for casual living-room recreation.*

But there is a problem: it takes a long time to play an infinite game!

Playing Muller Games in Finite Time

Robert McNaughton:

> *We believe that infinite games might have an interest for casual living-room recreation.*

But there is a problem: it takes a long time to play an infinite game! Thus:

- Scoring functions for Muller games.
- Use threshold score to obtain finite-duration variant.
- If threshold is large enough, obtain finite game with the same winning regions as infinite game.

Question

How large has the threshold to guarantee same winner?

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F\colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F\colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$								
$\mathrm{Acc}_{\{0\}}$								
$\mathrm{Sc}_{\{0,1\}}$								
$\mathrm{Acc}_{\{0,1\}}$								
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1							
$\mathrm{Acc}_{\{0\}}$	$\emptyset$							
$\mathrm{Sc}_{\{0,1\}}$								
$\mathrm{Acc}_{\{0,1\}}$								
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2						
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$						
$\mathrm{Sc}_{\{0,1\}}$								
$\mathrm{Acc}_{\{0,1\}}$								
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0					
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$					
$\mathrm{Sc}_{\{0,1\}}$								
$\mathrm{Acc}_{\{0,1\}}$								
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0				
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$				
$\mathrm{Sc}_{\{0,1\}}$								
$\mathrm{Acc}_{\{0,1\}}$								
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1			
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$			
$\mathrm{Sc}_{\{0,1\}}$								
$\mathrm{Acc}_{\{0,1\}}$								
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2		
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$		
$\mathrm{Sc}_{\{0,1\}}$								
$\mathrm{Acc}_{\{0,1\}}$								
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F : V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F : V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	
$\mathrm{Sc}_{\{0,1\}}$								
$\mathrm{Acc}_{\{0,1\}}$								
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$								
$\mathrm{Acc}_{\{0,1\}}$								
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0							
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$							
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0						
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$						
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0	1					
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$					
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1				
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$				
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2			
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$			
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F\colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F\colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2		
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$		
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F\colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F\colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$								
$\mathrm{Acc}_{\{0,1,2\}}$								

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F : V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F : V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0							
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$							

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0						
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$						

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0					
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$					

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0				
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$				

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0			
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$			

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F : V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F : V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0		
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$		

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F \colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F \colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\emptyset$

Scores and Accumulators

For $F \subseteq V$ define $\mathrm{Sc}_F\colon V^+ \to \mathbb{N}$ and $\mathrm{Acc}_F\colon V^+ \to 2^F$. Intuition:

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
- $\mathrm{Acc}_F(w)$: set $A \subset F$ of vertices (from F) seen since last increase or reset of Sc_F.

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0\}}$	1	2	0	0	1	2	0	0
$\mathrm{Acc}_{\{0\}}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\emptyset$

Remark

$F = \mathrm{Inf}(\rho) \Leftrightarrow \liminf_{n\to\infty} \mathrm{Sc}_F(\rho_0 \cdots \rho_n) = \infty$

Results

McNaughton's version: stop play when some Sc_F reaches $|F|! + 1$.

Theorem (McNaughton 2000)

Every Muller game and McNaughton's finite-time variant are won by the same player.

Results

McNaughton's version: stop play when some Sc_F reaches $|F|! + 1$.

Theorem (McNaughton 2000)

Every Muller game and McNaughton's finite-time variant are won by the same player.

Fearnley and Z.: stop play when some Sc_F reaches 3.

Theorem (Fearnley, Z. 2010)

Every Muller game and the variant up to score 3 are won by the same player.

Results

McNaughton's version: stop play when some Sc_F reaches $|F|! + 1$.

Theorem (McNaughton 2000)

Every Muller game and McNaughton's finite-time variant are won by the same player.

Fearnley and Z.: stop play when some Sc_F reaches 3.

Theorem (Fearnley, Z. 2010)

Every Muller game and the variant up to score 3 are won by the same player.

Stronger statement, which implies the theorem:

Lemma

If Player i wins the Muller game, then she can prevent her opponent from ever reaching a score of 3 for every set $F \in \mathcal{F}_{1-i}$.

Game Reductions

Reduce complicated game $\mathcal{G}$ to simpler game $\mathcal{G}'$: every play ρ in $\mathcal{G}$ is mapped (continuously) to play ρ' in $\mathcal{G}'$ that has the same winner.

$$\mathcal{G} \leq \mathcal{G}'$$
$$\rho \mapsto \rho'$$

Game Reductions

Reduce complicated game $\mathcal{G}$ to simpler game $\mathcal{G}'$: every play ρ in $\mathcal{G}$ is mapped (continuously) to play ρ' in $\mathcal{G}'$ that has the same winner.

$$\mathcal{G} \leq \mathcal{G}'$$
$$\rho \mapsto \rho'$$

Solving $\mathcal{G}'$ yields

- winner of $\mathcal{G}$ and
- corresponding finite-state winning strategy for winner.

Game Reductions

Reduce complicated game $\mathcal{G}$ to simpler game $\mathcal{G}'$: every play ρ in $\mathcal{G}$ is mapped (continuously) to play ρ' in $\mathcal{G}'$ that has the same winner.

$$\mathcal{G} \leq \mathcal{G}'$$
$$\rho \mapsto \rho'$$

Solving $\mathcal{G}'$ yields

- winner of $\mathcal{G}$ and
- corresponding finite-state winning strategy for winner.

Remark

Muller games cannot be reduced to safety games.

Reducing Muller Games to Safety Games

Recall: If Player i wins a Muller game, then she can prevent her opponent from ever reaching a score of 3 for every set $F \in \mathcal{F}_{1-i}$.

"Player 0 has a winning strategy iff she can prevent Player 1 from reaching a score of 3" $\Rightarrow$ safety condition!

Reducing Muller Games to Safety Games

Recall: If Player i wins a Muller game, then she can prevent her opponent from ever reaching a score of 3 for every set $F \in \mathcal{F}_{1-i}$.

"Player 0 has a winning strategy iff she can prevent Player 1 from reaching a score of 3" $\Rightarrow$ safety condition!

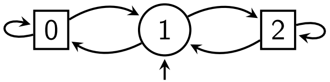
Construction:

- Ignore scores of Player 0.
- Identify plays having the same scores and accumulators for Player 1: $w =_{\mathcal{F}_1} w'$ iff $\mathrm{last}(w) = \mathrm{last}(w')$ and for all $F \in \mathcal{F}_1$:

$$\mathrm{Sc}_F(w) = \mathrm{Sc}_F(w') \text{ and } \mathrm{Acc}_F(w) = \mathrm{Acc}(w')$$

- Build $=_{\mathcal{F}_1}$-quotient of unravelling up to score 3 for Player 1.
- Winning condition for Player 0: avoid $\mathrm{Sc}_F = 3$ for all $F \in \mathcal{F}_1$.

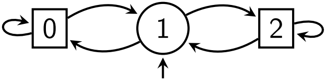
Continuing the Example



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Player 0 wins from 1 : move to 0 and 2 alternatingly.

Continuing the Example



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

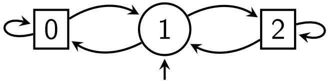
Player 0 wins from 1 : move to 0 and 2 alternatingly.

[0]: $\mathrm{Sc}_{\{0,1\}} = 0,\ \mathrm{Acc}_{\{0,1\}} = \{0\}$
$\mathrm{Sc}_{\{1,2\}} = 0,\ \mathrm{Acc}_{\{1,2\}} = \emptyset$

[1]: $\mathrm{Sc}_{\{0,1\}} = 0,\ \mathrm{Acc}_{\{0,1\}} = \{1\}$
$\mathrm{Sc}_{\{1,2\}} = 0,\ \mathrm{Acc}_{\{1,2\}} = \{1\}$

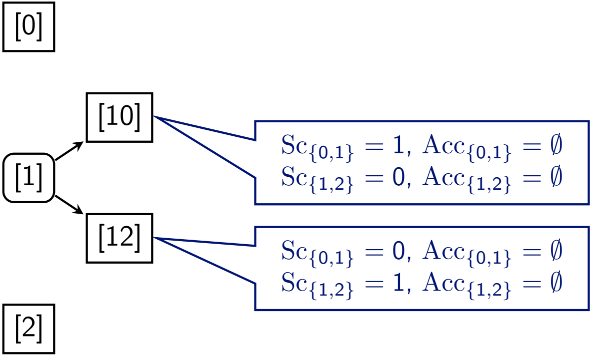
[2]: $\mathrm{Sc}_{\{0,1\}} = 0,\ \mathrm{Acc}_{\{0,1\}} = \emptyset$
$\mathrm{Sc}_{\{1,2\}} = 0,\ \mathrm{Acc}_{\{1,2\}} = \{2\}$

Continuing the Example

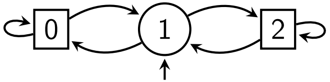


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Player 0 wins from 1 : move to 0 and 2 alternatingly.

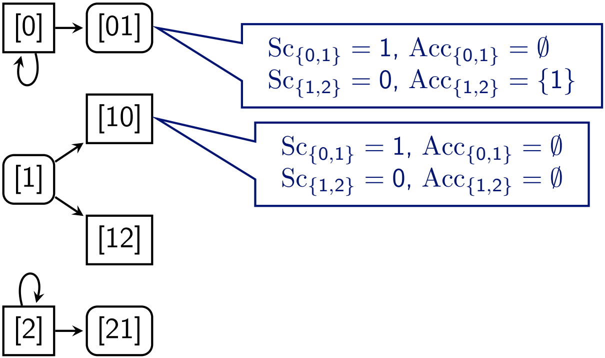


Continuing the Example

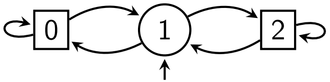


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Player 0 wins from 1 : move to 0 and 2 alternatingly.

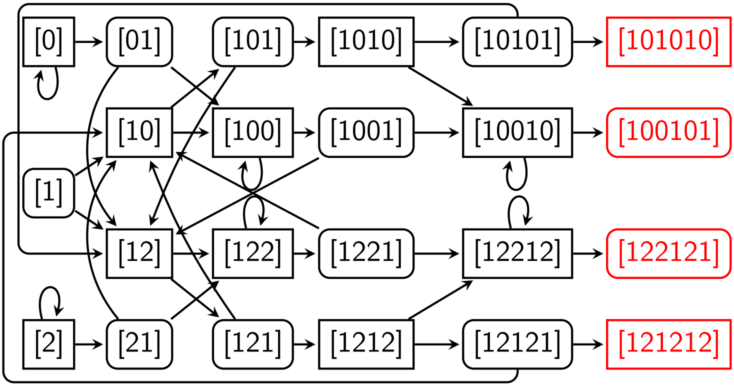


Continuing the Example

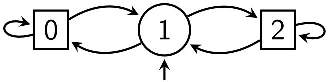


- $\mathcal{F}_0 = \{\{0, 1, 2\}, \{0\}, \{2\}\}$
- $\mathcal{F}_1 = \{\{0, 1\}, \{1, 2\}\}$

Player 0 wins from 1 : move to 0 and 2 alternatingly.

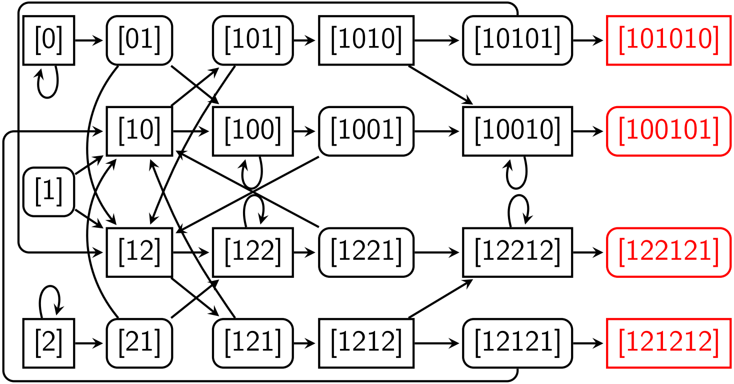


Continuing the Example

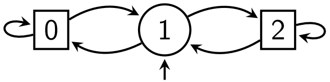


- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Player 0 wins from 1 : move to 0 and 2 alternatingly.

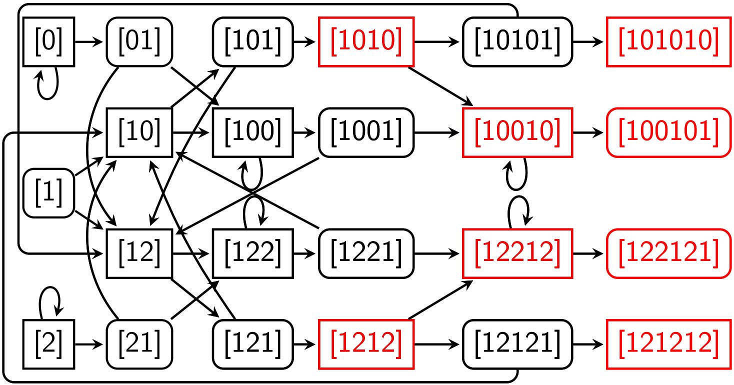


Continuing the Example



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Player 0 wins from 1 : move to 0 and 2 alternatingly.



Results

Theorem (Neider, Rabinovich, Z. 2011)

1. *Player i wins the Muller game from v iff she wins the safety game from $[v]_{=_{\mathcal{F}_1}}$.*
2. *Safety game can be turned into finite-state winning strategy for the Muller game.*
3. *Size of the safety game: $(n!)^3$.*

Results

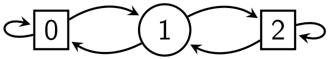
Theorem (Neider, Rabinovich, Z. 2011)

1. *Player i wins the Muller game from v iff she wins the safety game from $[v]_{=_{\mathcal{F}_1}}$.*
2. *Safety game can be turned into finite-state winning strategy for the Muller game.*
3. *Size of the safety game: $(n!)^3$.*

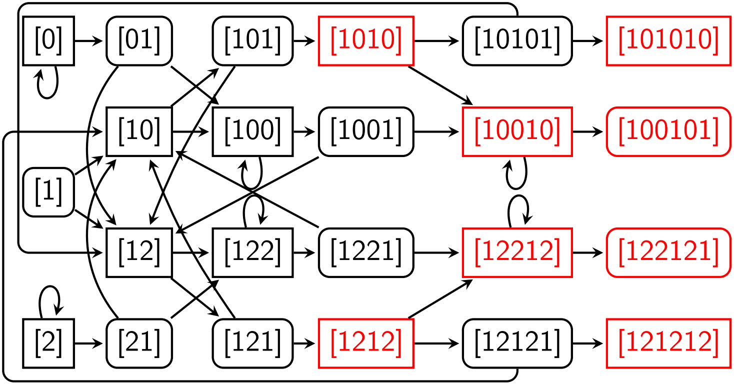
Remarks:

- Size of parity game in LAR-reduction $n!$. But: simpler algorithms for safety games.
- 2. does not hold for Player 1.
- Not a reduction in the classical sense: not every play of the Muller game can be mapped to a play in the safety game.

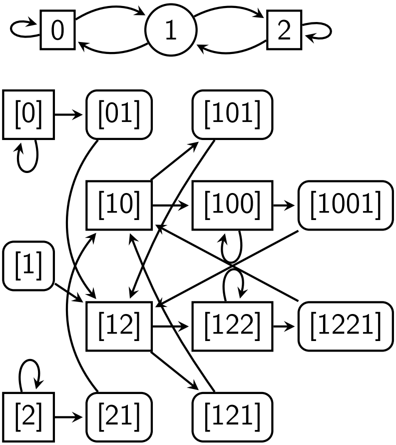
Proof Idea: Safety to Muller



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$



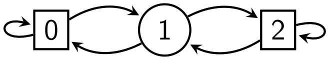
Proof Idea: Safety to Muller



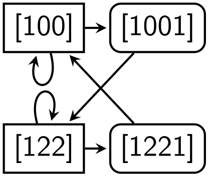
- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$

Pick a winning strategy for the safety game. This "is" a finite-state winning strategy for the Muller game.

Proof Idea: Safety to Muller



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
- $\mathcal{F}_1 = \{\{0,1\},\{1,2\}\}$



Even better: only use "maximal" elements, yields smaller memory.

Safety Reductions

Definition

$\mathcal{G} = (\mathcal{A}, \mathrm{Win})$ with vertex set V is safety reducible, if there is a regular $L \subseteq V^*$ such that:

- For every $\rho \in V^\omega$: if $\mathrm{Pref}(\rho) \subseteq L$, then $\rho \in \mathrm{Win}$.
- If $v \in W_0(\mathcal{G})$, then Player 0 has a strategy σ with $\mathrm{Pref}(\rho) \subseteq L$ for every ρ consistent with σ and starting in v.

Safety Reductions

Definition

$\mathcal{G} = (\mathcal{A}, \mathrm{Win})$ with vertex set V is safety reducible, if there is a regular $L \subseteq V^*$ such that:

- For every $\rho \in V^\omega$: if $\mathrm{Pref}(\rho) \subseteq L$, then $\rho \in \mathrm{Win}$.
- If $v \in W_0(\mathcal{G})$, then Player 0 has a strategy σ with $\mathrm{Pref}(\rho) \subseteq L$ for every ρ consistent with σ and starting in v.

Theorem (Neider, Rabinovich, Z. 2011)

$\mathcal{G}$ safety reducible with $L(\mathfrak{A}) \subseteq V^*$ for DFA $\mathfrak{A} = (Q, V, q_0, \delta, F)$. Define the safety game $\mathcal{G}_S = (\mathcal{A} \times \mathfrak{A}, V \times F)$. Then:

1. *Player i wins $\mathcal{G}$ from v if and only if Player i wins $\mathcal{G}_S$ from $(v, \delta(q_0, v))$.*
2. *Player 0 has a finite-state winning strategy for $\mathcal{G}$ with memory states Q (if she wins $\mathcal{G}$).*

Safety Reductions: Applications

- Reachability games: reach F after $|V \setminus F|$ steps.
- Büchi games: reach F every $|V \setminus F|$ steps.
- co-Büchi games: avoid visiting $v \in V \setminus F$ twice.
- Request-response games and poset games: bound waiting times (Horn, Thomas, Wallmeier 2008; Z. 2009).
- parity, Rabin, Streett games: progress measure algorithms "are" safety reductions (Jurdziński 2000; Piterman, Pnueli 2006).
- Muller games: bound scores.

If you can solve safety games, you can solve all these games.
Caveat: safety games will be larger than original game.