Visibly Linear Dynamic Logic

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The Everlasting Quest for Expressiveness

Consider an arbiter granting access to a shared resource.

Requirements:

- "Every request q is eventually answered by a response p"
- "Every request q is eventually answered by a response p after an *even* number of steps"
- "There are never more responses than requests"

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  Expressible with pushdown automata/context-free grammars as guards $\Rightarrow$ Visibly Linear Dynamic Logic

Outline

1. **Preliminaries**

2. **Expressiveness**

3. **VLDL Verification**

4. **Discussion**

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Visibly Pushdown Automata

Partition input alphabet Σ into Σ_c (calls), Σ_r (returns), and Σ_ℓ (local actions).

A visibly pushdown automaton (VPA) has to

- push when processing a call,
- pop when processing a return, and
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Stack height determined by input word $\Rightarrow$ closure under union, intersection, and complement.

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Examples:

- $a^n b^n$ is a VPL, if a is a call and b a return.
- ww^R is not a VPL.

Visibly Linear Dynamic Logic (VLDL)

Syntax

$$\varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \langle\mathfrak{A}\rangle\varphi \mid [\mathfrak{A}]\varphi$$

where $p \in P$ ranges over atomic propositions and $\mathfrak{A}$ ranges over VPA's. All VPA's have the same partition of 2^P into calls, returns, and local actions.

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Semantics: $(w \in (2^P)^\omega)$

- $w \models \langle\mathfrak{A}\rangle\varphi$ if there exists an n such that $w_0 \cdots w_{n-1}$ is accepted by $\mathfrak{A}$ and $w_n w_{n+1} w_{n+2} \cdots \models \varphi$.
- $w \models [\mathfrak{A}]\varphi$ if for every n s.t. $w_0 \cdots w_{n-1}$ is accepted by $\mathfrak{A}$ we have $w_n w_{n+1} w_{n+2} \cdots \models \varphi$.

Example

"Every request q is eventually answered by a response p and there are never more responses than requests"

$$[\mathfrak{A}^*](\, q \rightarrow \langle \mathfrak{A}^* \rangle p\,) \wedge \neg \langle \mathfrak{A} \rangle \mathtt{true}$$

where

- $\mathfrak{A}^*$ accepts every word, and
- $\mathfrak{A}$ accepts those words with more responses than requests.

Both languages are visibly pushdown, if

- $\{q\}$ is a call,
- $\{p\}$ is a return, and
- $\emptyset$ and $\{p, q\}$ are local actions.

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Expressiveness

Lemma

VLDL and non-deterministic ω-VPA are expressively equivalent.

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Proof Idea

VLDL

non-deterministic
ω-VPA

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VLDL

Deterministic Stair Automata

$\mathcal{O}(2^n)$

[LMS '04]

non-deterministic ω-VPA

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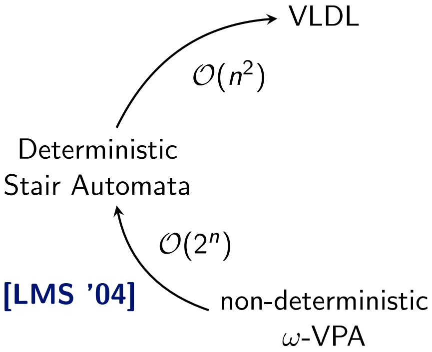

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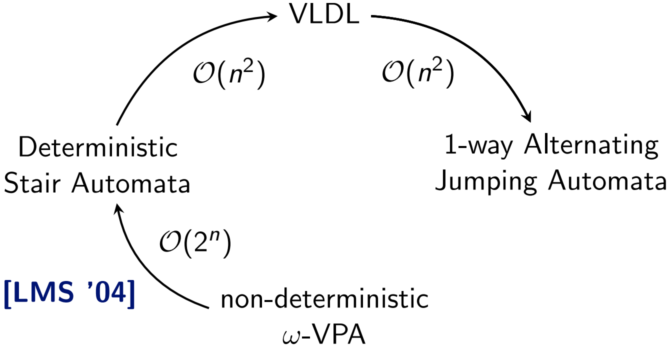

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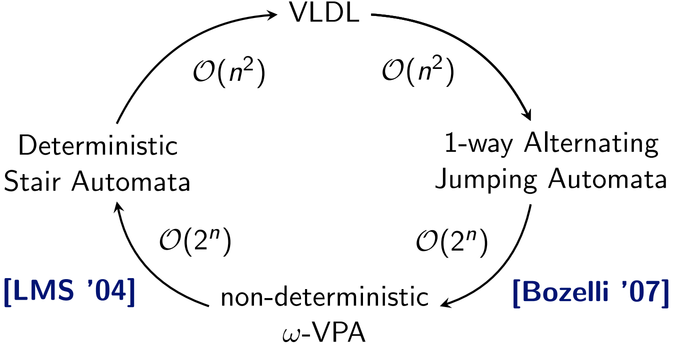

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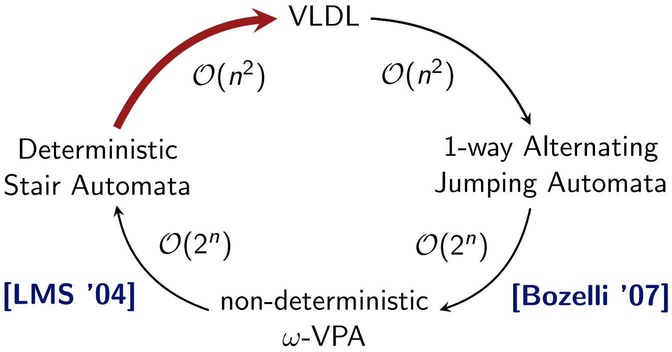
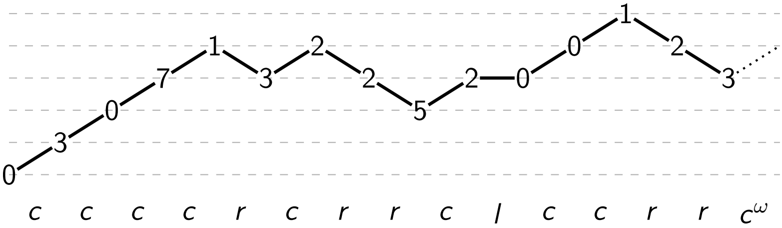


From Stair Automata to VLDL



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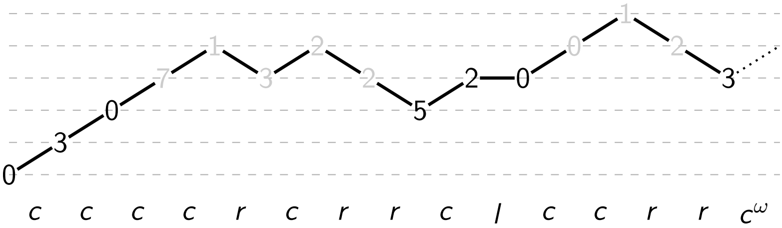

Acceptance: maximal priority occuring at infinitely many steps even

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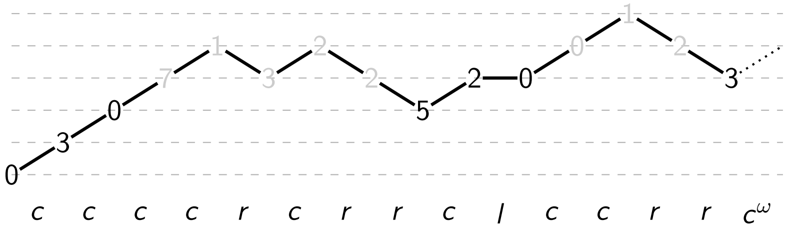

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Equivalently: For some state q of even priority c there is step with state q s.t.

1. after this step, no larger priority appears at a step, and
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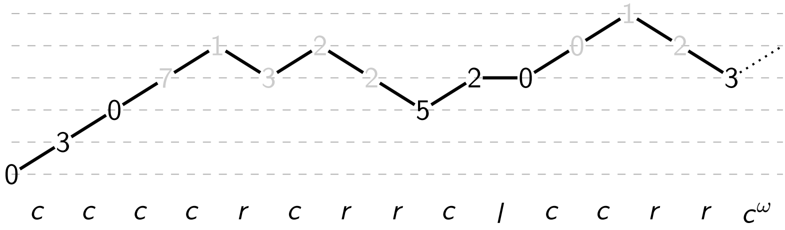

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$$\bigvee_{q \in Q_{even}} \langle {}_{q_I}\mathfrak{A}'_q \rangle \left(\bigwedge_{q' \in Q_{>\Omega(q)}} [{}_q\mathfrak{A}'_{q'}]\texttt{false} \right) \wedge [\mathfrak{A}'_q]\langle {}_q\mathfrak{A}'_q \rangle \texttt{true}$$

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Satisfiability

Theorem

VLDL Satisfiability is EXPTIME*-complete.*

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Proof Sketch

- **Membership:** Construct equivalent ω-VPA and check it for emptiness.
- **Hardness:** Adapt EXPTIME-hardness proof of LTL model-checking of pushdown systems **[BEM '97]**

Model Checking

Theorem

VLDL model checking of visibly pushdown systems is EXPTIME*-complete.*

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Proof Sketch

- **Membership:** To check $\mathcal{S} \models \varphi$, construct ω-VPA equivalent to $\neg\varphi$ and check intersection with $\mathcal{S}$ for emptiness.
- **Hardness:** Follows immediately from EXPTIME-hardness of satisfiability.

Synthesis

Theorem

Solving infinite games on visibly pushdown graphs with VLDL winning conditions is $3\textsc{ExpTime}$*-complete.*

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Solving infinite games on visibly pushdown graphs with VLDL winning conditions is $3\textsc{ExpTime}$-complete.

Proof Sketch

- **Membership:** To determine the winner, construct an ω-VPA that accepts the winning condition and solve the resulting game with VPA winning condition **[LMS '04]**.
- **Hardness:** Adapt $3\textsc{ExpTime}$-hardness proof of pushdown games with LTL winning condition **[LMS '04]**.

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"If p holds true immediately after entering module m, it shall hold immediately after the corresponding return from m as well"

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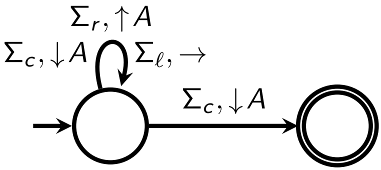
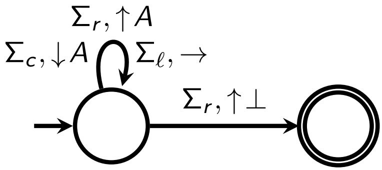
VLDL:

$$[\mathfrak{A}_c](p \rightarrow \langle \mathfrak{A}_r \rangle p)$$

with



$\mathfrak{A}_c$



$\mathfrak{A}_r$

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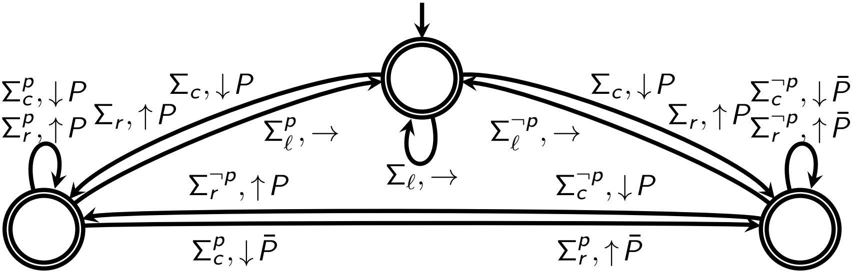
ω-VPA:



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VLTL: [Bozzelli '14]

$$(\alpha; \texttt{true})|\alpha\rangle\texttt{false}$$

with visibly rational expression α below:

$$[(p \cup q)^* \texttt{call}_m \, [(q\Box) \cup (p\Box p)] \, \texttt{return}_m (p \cup q)^*]^{\circlearrowleft\Box} \curvearrowleft_{\Box} (p \cup q)^*$$

Conclusion

Results:

- VLDL as expressive as ω-VPA

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	validity	model-checking	infinite games
LTL	PSPACE	PSPACE	2EXPTIME
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LTL	PSpace	PSpace	2ExpTime
LDL	PSpace	PSpace	2ExpTime
VLDL	ExpTime	ExpTime	3ExpTime

Conclusion

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VLTL	EXPTIME	EXPTIME	?

- Using (deterministic) pushdown automata as guards leads to undecidability, i.e.,

$$\langle \mathfrak{A}_1 \rangle \# \wedge \langle \mathfrak{A}_2 \rangle \# \wedge \text{“exactly one } \#\text{”}$$

is satisfiable $\Leftrightarrow L(\mathfrak{A}_1) \cap L(\mathfrak{A}_2) \neq \emptyset$.