Prompt and Parametric LTL Games

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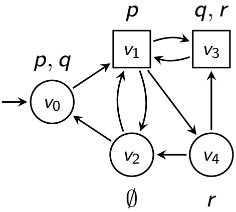
Outline

1. **Introduction**
2. Parametric LTL
3. Conclusion

Infinite Games

Played in finite arena $\mathcal{A} = (V, V_0, V_1, E, v_0, l)$ with labeling $l \colon V \to 2^P$. Winning conditions are expressed in extensions of LTL over P.



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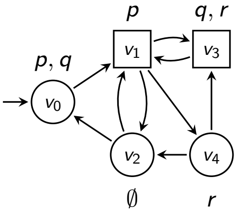

Theorem (Pnueli, Rosner '89)

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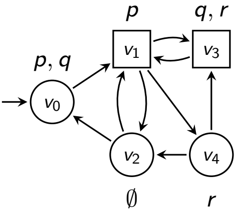

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However, LTL lacks capabilities to express timing constraints.

There are many extensions of LTL to overcome this. Here, we consider two of them:

- PLTL: Parametric LTL (Alur et. al., '99)
- $\mathrm{PROMPT-LTL}$ (Kupferman et. al., '07)

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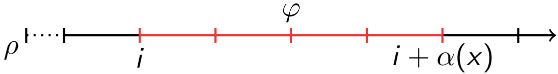
PLTL

Let $\mathcal{X}$ and $\mathcal{Y}$ two disjoint sets of variables. Add $\mathbf{F}_{\leq x}$ for $x \in \mathcal{X}$ and $\mathbf{G}_{\leq y}$ for $y \in \mathcal{Y}$ to LTL. Semantics defined w.r.t. variable valuation $\alpha \colon \mathcal{X} \cup \mathcal{Y} \to \mathbb{N}$.

$(\rho, i, \alpha) \models \mathbf{F}_{\leq x}\varphi$:



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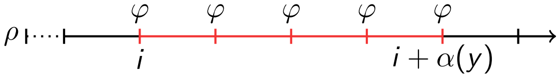

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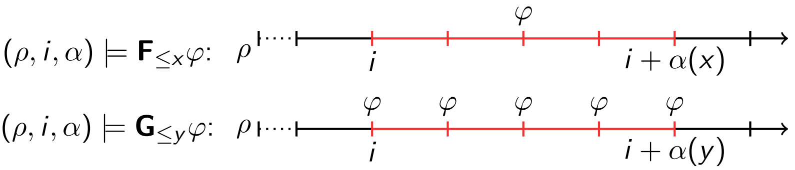


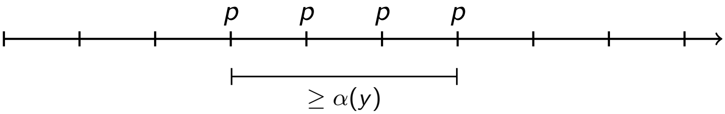
PLTL game $(\mathcal{A}, \varphi)$:

- σ winning strategy for Player 0 w.r.t. α iff for all plays ρ consistent with σ: $(\rho, 0, \alpha) \models \varphi$.
- τ winning strategy for Player 1 w.r.t. α iff for all plays ρ consistent with τ: $(\rho, 0, \alpha) \not\models \varphi$.
- $\mathcal{W}^i_{\mathcal{G}} = \{\alpha \mid \text{Player } i \text{ has winning strategy for } \mathcal{G} \text{ w.r.t. } \alpha\}$.

PLTL Games: Examples

Winning condition $\mathbf{FG}_{\leq y}\, p$:

- Player 0's goal: eventually satisfy p for at least $\alpha(y)$ steps.



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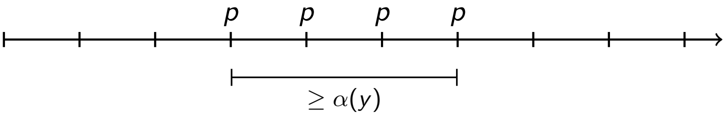
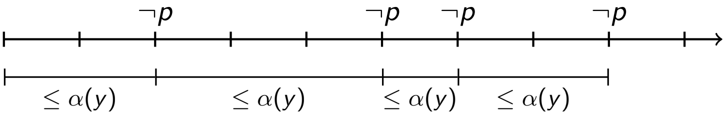


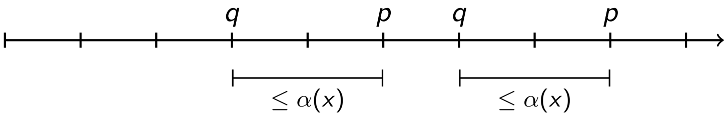
- Player 1's goal: reach vertex with $\neg p$ at least every $\alpha(y)$ steps.



PLTL Games: Examples

Winning condition $\mathbf{G}(q \rightarrow \mathbf{F}_{\leq x} p)$: "Every request q is eventually responded by p".

- Player 0's goal: uniformly bound the waiting times between requests q and responses p by $\alpha(x)$.



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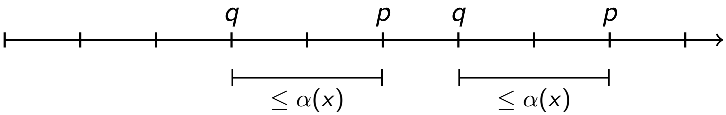
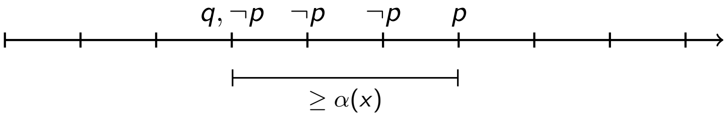


- Player 1's goal: either request q and prevent response p or enforce waiting time greater than $\alpha(x)$.



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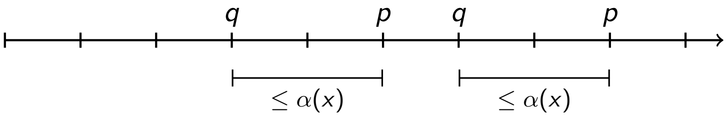

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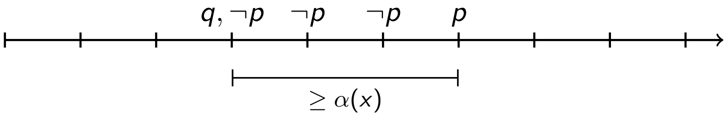

Note: both winning conditions induce an optimization problem: maximize $\alpha(y)$ resp. minimize $\alpha(x)$.

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Theorem

Let $\mathcal{G}$ be a $\mathrm{PROMPT-LTL}$ game. The emptiness problem for $\mathcal{W}_{\mathcal{G}}^{0}$ is **2EXPTIME** *complete.*

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Let $\mathcal{G}$ be a $\mathrm{PROMPT-LTL}$ game. The emptiness problem for $\mathcal{W}^0_{\mathcal{G}}$ is **2EXPTIME** *complete.*

Proof

- **2EXPTIME** algorithm: apply *alternating-color technique* of Kupferman et al.. Reduce $\mathcal{G}$ to an LTL game $\mathcal{G}'$ such that a finite-state winning strategy for $\mathcal{G}'$ can be transformed into a winning strategy for $\mathcal{G}$ that bounds the waiting times.
- **2EXPTIME** hardness follows from **2EXPTIME** hardness of solving LTL games.

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Theorem

Let $\mathcal{G}$ be a PLTL game. The emptiness, finiteness, and universality problem for $\mathcal{W}^i_{\mathcal{G}}$ are **2EXPTIME***-complete.*

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Proof

- **2EXPTIME** algorithms: Emptiness for formulae with only $\mathbf{F}_{\leq x}$: reduction to $\mathrm{PROMPT}-\mathrm{LTL}$ games. For the full logic and the other problems use:
  - Duality of $\mathbf{F}_{\leq x}$ and $\mathbf{G}_{\leq y}$.
  - Monotonicity of $\mathbf{F}_{\leq x}$ and $\mathbf{G}_{\leq y}$.
- **2EXPTIME** hardness follows from **2EXPTIME** hardness of solving LTL games.

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If φ contains only $\mathbf{F}_{\leq x}$ respectively only $\mathbf{G}_{\leq y}$, then solving games is an optimization problem: which is the *best* valuation in $\mathcal{W}_{\mathcal{G}}^{0}$?

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- *Determine* $\min_{\alpha \in \mathcal{W}^0_{\mathcal{G}_{\mathbf{F}}}} \max_{x \in \mathrm{var}(\varphi_{\mathbf{F}})} \alpha(x)$.

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We considered infinite games with winning conditions in extensions of LTL with bounded temporal operators.

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Further research:

- Better algorithms for the optimization problems.
- Hardness results for the optimization problems.
- Tradeoff between size and quality of a finite-state strategy.
- Time-optimal winning strategies for other winning conditions.