Optimal Bounds in Parametric LTL Games

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Motivation

Parametric temporal logic (PLTL, **[Alur et. al., '99]**):

- LTL with $\mathbf{F}_{\leq x}$, $\mathbf{G}_{\leq y}$.
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- Semantics w.r.t. variable valuation.

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Results:

- Gasics Meeting Aachen (2009): determining whether Player 0 wins a PLTL game w.r.t. some, infinitely many, or all variable valuations is **2EXPTIME**-complete.
- Today: determining optimal variable valuations that let Player 0 win a PLTL game can be computed in doubly-exponential time.

Outline

1. **Introduction**
2. Results
3. Proof Sketch
4. Conclusion

Parametric LTL

LTL:

$$\varphi ::= p \mid \neg p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \mathbf{X}\varphi \mid \varphi \mathbf{U} \varphi \mid \varphi \mathbf{R} \varphi$$

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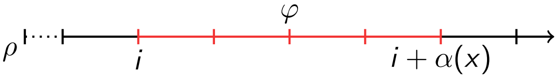

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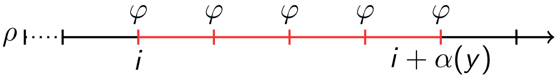

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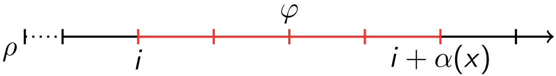

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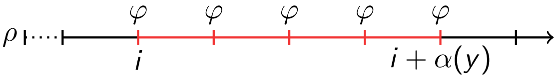

The operators $\mathbf{U}_{\leq x}$, $\mathbf{R}_{\leq y}$, $\mathbf{F}_{>y}$, $\mathbf{G}_{>x}$, $\mathbf{U}_{>y}$, and $\mathbf{R}_{>x}$ (with the obvious semantics) are syntactic sugar, and will be ignored.

Infinite Games

An arena $\mathcal{A} = (V, V_0, V_1, E, v_0, l)$ consists of

- a finite, directed graph (V, E),
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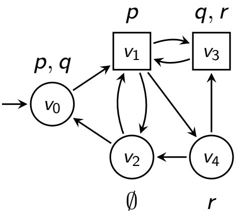
- an initial vertex v_0,
- a labeling $l \colon V \to 2^P$ for some set P of atomic propositions.



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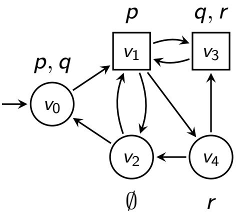

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- Play: path $\rho_0\rho_1\rho_2\ldots$ through (V, E) starting in v_0.
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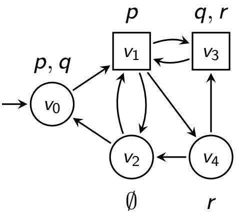

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- $\rho_0\rho_1\rho_2\ldots$ winning for Player 0 w.r.t. variable valuation α: $(\rho_0\rho_1\rho_2\ldots, 0, \alpha) \models \varphi$. Otherwise winning for Player 1.
- Strategy for Player i: $\sigma\colon V^*V_i \to V$ s.t. $(v, \sigma(wv)) \in E$.
- Winning strategy for Player i w.r.t. α: every play that is consistent with σ is won by Player i.

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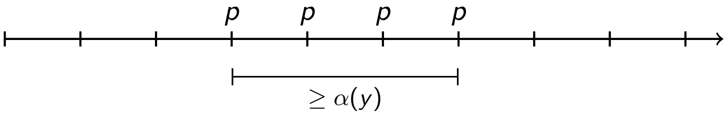
- Winning condition $\mathbf{FG}_{\leq y} p$. Player 0's goal: eventually satisfy p for at least $\alpha(y)$ steps.



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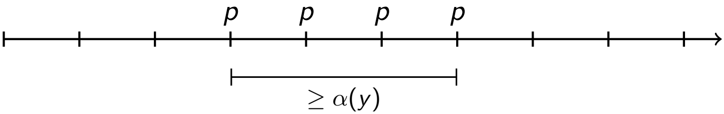
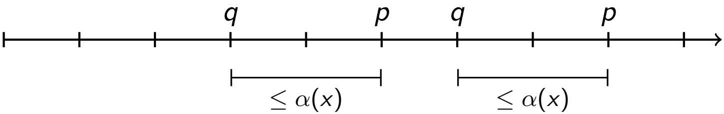


- Winning condition $\mathbf{G}(q \rightarrow \mathbf{F}_{\leq x} p)$. Player 0's goal: uniformly bound the waiting times between requests q and responses p by $\alpha(x)$.



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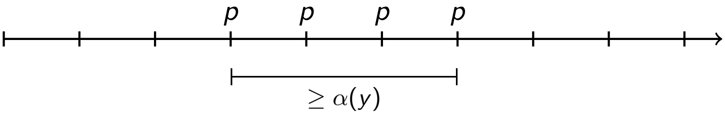

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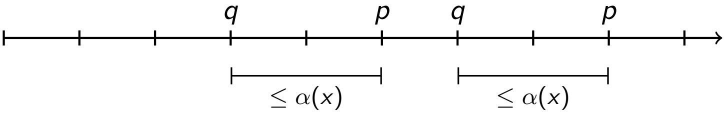

Note: both winning conditions induce an optimization problem: maximize $\alpha(y)$ respectively minimize $\alpha(x)$.

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Solving PLTL Games

Theorem (Pnueli, Rosner '89)

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The set of winning valuations for Player i in a PLTL game $\mathcal{G}$ is

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Theorem

The following problems are **2EXPTIME**-*complete: Given* $\mathcal{G}$ *and* i*:*

i) *Is* $\mathcal{W}^i_{\mathcal{G}}$ *non-empty?*

ii) *Is* $\mathcal{W}^i_{\mathcal{G}}$ *infinite?*

iii) *Is* $\mathcal{W}^i_{\mathcal{G}}$ *universal?*

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Let $\varphi_{\mathbf{F}}$ be $\mathbf{G}_{\leq y}$-free and $\varphi_{\mathbf{G}}$ be $\mathbf{F}_{\leq x}$-free, let $\mathcal{G}_{\mathbf{F}} = (\mathcal{A}, \varphi_{\mathbf{F}})$ and $\mathcal{G}_{\mathbf{G}} = (\mathcal{A}, \varphi_{\mathbf{G}})$. The following values can be computed in doubly-exponential time:

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Duality and monotonicity: it suffices to determine $\min_{\alpha \in \mathcal{W}^0_{\mathcal{G}_\mathsf{F}}} \max_{x \in \mathrm{var}(\varphi_\mathsf{F})} \alpha(x)$.

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There exists a $k \in \mathcal{O}(|\mathcal{A}| \cdot 2^{2^{|\varphi_\mathsf{F}|}})$ *such that*

$$\mathcal{W}^0_{\mathcal{G}_\mathsf{F}} \neq \emptyset \iff x \mapsto k \in \mathcal{W}^0_{\mathcal{G}_\mathsf{F}} \iff \min_{\alpha \in \mathcal{W}^0_{\mathcal{G}_\mathsf{F}}} \max_{x \in \mathrm{var}(\varphi_\mathsf{F})} \alpha(x) \leq k \enspace .$$

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As we can test $\alpha \in \mathcal{W}^0_{\mathcal{G}_\mathbf{F}}$ effectively, it suffices to check all $k' < k$.

Example: $\varphi_\mathbf{F} = \mathbf{G}(q \rightarrow \mathbf{F}_{\leq x} p)$ and $\alpha(x) = 2$:

$$\alpha \in \mathcal{W}^0_{\mathcal{G}_\mathbf{F}} \iff \text{Player 0 wins } (\mathcal{A}, \mathbf{G}(q \rightarrow p \vee \mathbf{X}(p \vee \mathbf{X}p))) \ .$$

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Problem: this approach takes quadruply-exponential time.

A Better Idea

Faster algorithm for "$\alpha \in \mathcal{W}^0_{\mathcal{G}_{\mathbf{F}}}$?" provided $\alpha(x) \leq k$ for all $x \in \mathrm{var}(\varphi_{\mathbf{F}})$:

1. Replace all $\mathbf{F}_{\leq x}$ by $\mathbf{F}$ to obtain φ'.
2. Build Büchi automaton $\mathfrak{A}_{\varphi'}$.
3. Determinize $\mathfrak{A}_{\varphi'}$ and add counters simulating α to obtain deterministic parity automaton $\mathfrak{P}$.
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So, we have to solve exponentially many parity games, each in doubly-exponential time: gives doubly-exponential time.

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2. Build Büchi automaton $\mathfrak{A}_{\varphi'}$ (textbook method).

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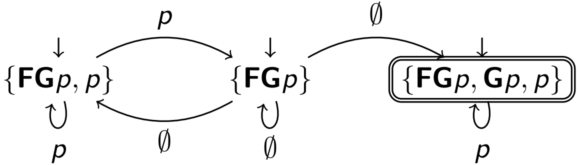

Accepting run: visit accepting state every $\alpha(x)$ transitions.

In general: one set of final states F_x for every $x \in \mathrm{var}(\varphi_{\mathbf{F}})$ (generalized Büchi).

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We have presented an algorithm to determine optimal bounds in PLTL games in doubly-exponential time.

- For a known (doubly-exponential) upper bound k we test all smaller values $k' < k$.
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Open question:
Is there a *direct* algorithm that avoids checking all $k' < k$?