Down the Borel Hierarchy: Solving Muller Games via Safety Games

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Introduction

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We present a direct construction from Muller to safety with only exponential blowup $\Rightarrow$ new algorithm, new memory structure, and permissive strategies for Muller games.

Muller Games

Muller games $(\mathcal{A}, \mathcal{F}_0, \mathcal{F}_1)$:

- arena $\mathcal{A}$ and partition $(\mathcal{F}_0, \mathcal{F}_1)$ containing the loops of $\mathcal{A}$.
- Player i wins ρ iff $\mathrm{Inf}(\rho) = \{v \mid \exists^\omega n \text{ s.t. } \rho_n = v\} \in \mathcal{F}_i$.

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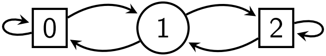
Running example



- $\mathcal{F}_0 = \{\{0, 1, 2\}, \{0\}, \{2\}\}$
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Player 0 has a winning strategy from every vertex: alternate between 0 and 2. This requires two memory states.

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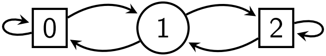

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Theorem

Muller games are determined with finite-state strategies of size $n!$.

Outline

1. **Scoring Functions for Muller Games**
2. Solving Muller Games by Solving Safety Games
3. Conclusion

Scoring Functions

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$$\mathrm{Sc}_F(v) = \begin{cases} 1 & \text{if } F = \{v\}, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\mathrm{Acc}_F(v) = \begin{cases} \emptyset & \text{if } F = \{v\}, \\ F \cap \{v\} & \text{otherwise.} \end{cases}$$

Scoring Functions

Let $F \subseteq V$, $F \neq \emptyset$. For $v \in V$ and $w \in V^+$ define

$$\mathrm{Sc}_F(wv) = \begin{cases} 0 & \text{if } v \notin F, \\ \mathrm{Sc}_F(w) & \text{if } v \in F \wedge \mathrm{Acc}_F(w) \neq (F \setminus \{v\}), \\ \mathrm{Sc}_F(w) + 1 & \text{if } v \in F \wedge \mathrm{Acc}_F(w) = (F \setminus \{v\}), \end{cases}$$

and

$$\mathrm{Acc}_F(wv) = \begin{cases} \emptyset & \text{if } v \notin F, \\ \mathrm{Acc}_F(w) \cup \{v\} & \text{if } v \in F \wedge \mathrm{Acc}_F(w) \neq (F \setminus \{v\}), \\ \emptyset & \text{if } v \in F \wedge \mathrm{Acc}_F(w) = (F \setminus \{v\}). \end{cases}$$

Scoring Functions cont'd

- $\mathrm{Sc}_F(w)$: maximal $k \in \mathbb{N}$ such that F is visited k times since last vertex in $V \setminus F$ (reset).
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Example:

w	0	0	1	1	0	0	1	2
$\mathrm{Sc}_{\{0,1\}}$	0	0	1	1	2	2	3	0
$\mathrm{Acc}_{\{0,1\}}$	$\{0\}$	$\{0\}$	$\emptyset$	$\{1\}$	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	

Scoring Functions cont'd

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$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\emptyset$

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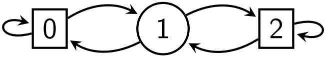
$\mathrm{Sc}_{\{0,1,2\}}$	0	0	0	0	0	0	0	1
$\mathrm{Acc}_{\{0,1,2\}}$	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\emptyset$

Remark

$F = \mathrm{Inf}(\rho) \Leftrightarrow \liminf_{n\to\infty} \mathrm{Sc}_F(\rho_0 \cdots \rho_n) = \infty$

Two Examples



- $\mathcal{F}_0 = \{\{0, 1, 2\}, \{0\}, \{2\}\}$
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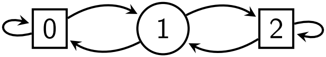
Two Examples



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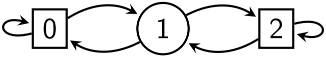
Losing player (Player 1) can enforce score of two:

Two Examples



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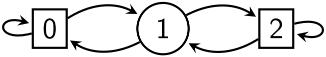
1

Two Examples



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$$1 \longrightarrow 2 \qquad \text{(w.l.o.g.)}$$

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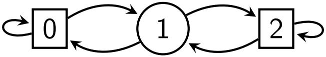

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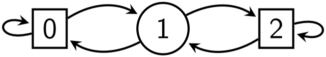

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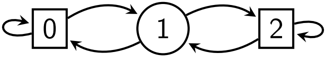

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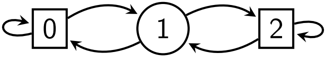

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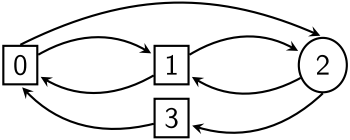

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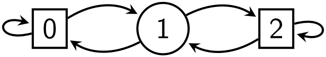

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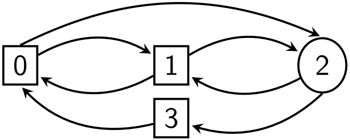

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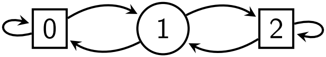
Losing player (Player 1) is the first to reach a score of two:

Two Examples



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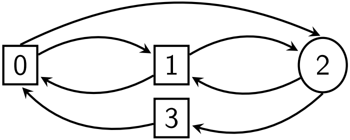

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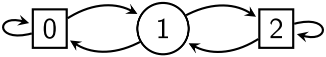
3

Two Examples



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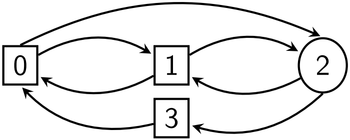

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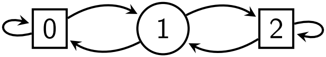
Losing player (Player 1) is the first to reach a score of two:

$$3 \longrightarrow 0$$

Two Examples



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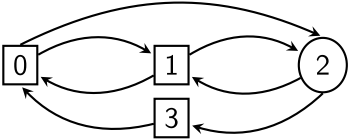

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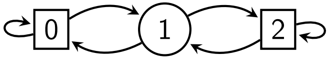
Two Examples



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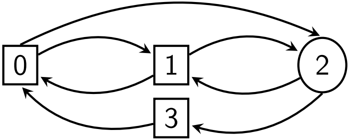

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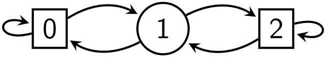
Two Examples



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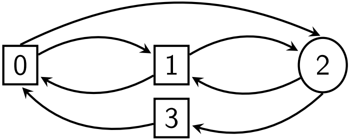

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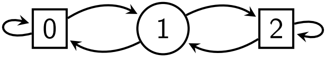
Losing player (Player 1) is the first to reach a score of two:

$$\begin{tikzcd} & & & 1 \arrow[r] & 0 \\ 3 \arrow[r] & 0 \arrow[r] & 2 \arrow[ur] & & \end{tikzcd}$$

Two Examples



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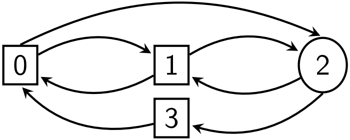

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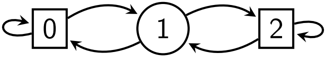

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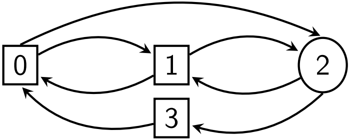

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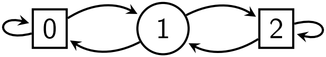

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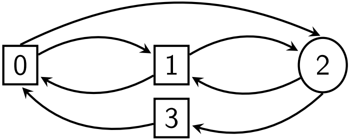

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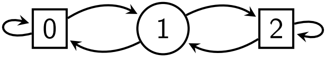

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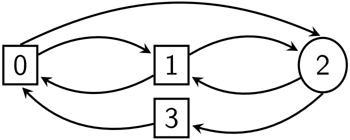

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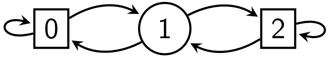

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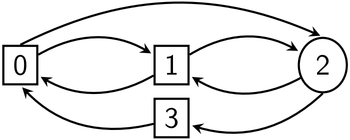

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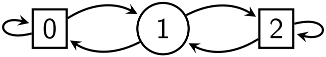

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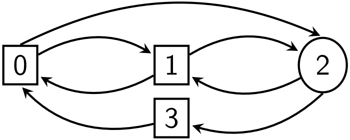

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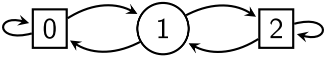

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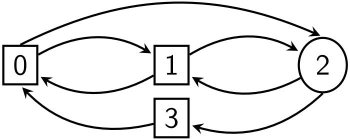

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Losing player (Player 1) is the first to reach a score of two:

$$\begin{tikzcd} & & & 1 \arrow[r] & 0 \arrow[r] & 1 \arrow[r] & 2 \\ 3 \arrow[r] & 0 \arrow[r] & 2 \arrow[ur] \arrow[dr] & & & & \\ & & & 3 \arrow[r] & 0 \arrow[r] & 2 & \end{tikzcd}$$

$\mathrm{Sc}_{\{0,1,2\}} = 2$

$\mathrm{Sc}_{\{0,2,3\}} = 2$

Results

Lemma (FZ10)

On her winning region, Player i can prevent her opponent from ever reaching a score of 3 for every set $F \in \mathcal{F}_{1-i}$.

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Corollary

Two "reductions": Muller game to..

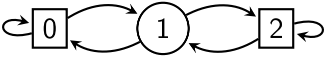
1. *..reachability game on unraveling up to score* 3*: yields winning regions, but no winning strategies.*
2. *..safety game: see next slides: yields winning regions and one winning strategy.*

Remember: winning regions and one winning strategy is the best we can hope for.

Outline

1. Scoring Functions for Muller Games
2. **Solving Muller Games by Solving Safety Games**
3. Conclusion

“Reducing“ Muller Games to Safety Games



- $\mathcal{F}_0 = \{\{0,1,2\},\{0\},\{2\}\}$
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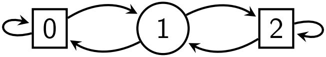
“Reducing“ Muller Games to Safety Games



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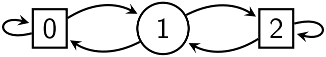
Idea: track of Player 1's scores and avoid $\mathrm{Sc}_F = 3$ for $F \in \mathcal{F}_1$.

“Reducing“ Muller Games to Safety Games



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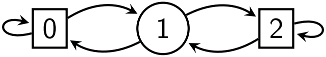
Idea: track of Player 1’s scores and avoid $\mathrm{Sc}_F = 3$ for $F \in \mathcal{F}_1$.

- Ignore scores of Player 0.
- Identify plays having the same scores and accumulators for Player 1: $w =_{\mathcal{F}_1} w'$ iff $\mathrm{last}(w) = \mathrm{last}(w')$ and for all $F \in \mathcal{F}_1$:

  $$\mathrm{Sc}_F(w) = \mathrm{Sc}_F(w') \text{ and } \mathrm{Acc}_F(w) = \mathrm{Acc}(w')$$

- Build $=_{\mathcal{F}_1}$-quotient of unravelling up to score 3 for Player 1.
- Winning condition for Player 0: avoid $\mathrm{Sc}_F = 3$ for all $F \in \mathcal{F}_1$.

"Reducing" Muller Games to Safety Games



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[0]

$\mathrm{Sc}_{\{0,1\}} = 0$, $\mathrm{Acc}_{\{0,1\}} = \{0\}$
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[2]

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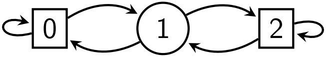

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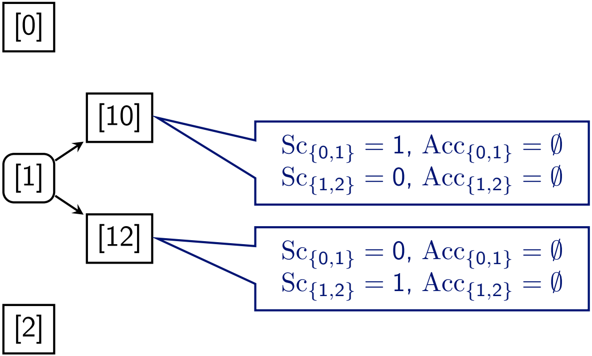

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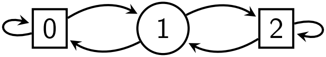

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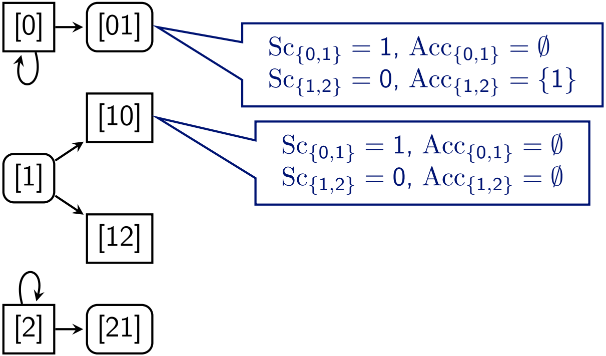

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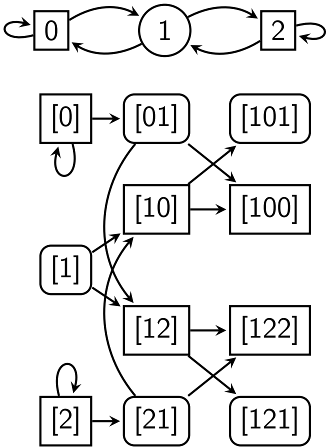

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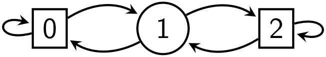

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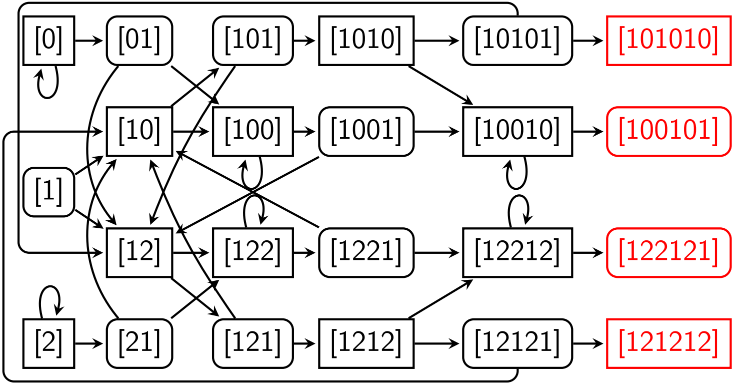

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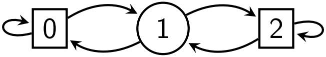

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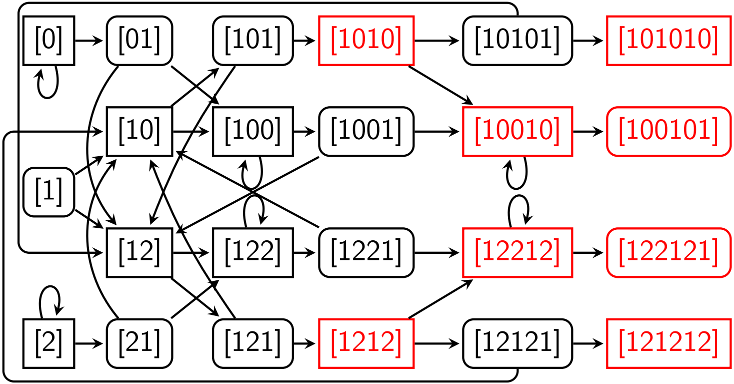

Results

Theorem (NRZ11)

1. *Player i wins the Muller game from v iff she wins the safety game from $[v]_{=_{\mathcal{F}_1}}$.*
2. *Player 0's winning region in the safety game can be turned into finite-state winning strategy for her in the Muller game.*
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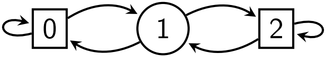
Remarks:

- Size of parity game in LAR-reduction $n!$. But: safety games allow much simpler algorithms.
- 2. does not hold for Player 1.
- Not a reduction in the classical sense: not every play of the Muller game can be mapped to a play in the safety game.

The Proof: Safety to Muller



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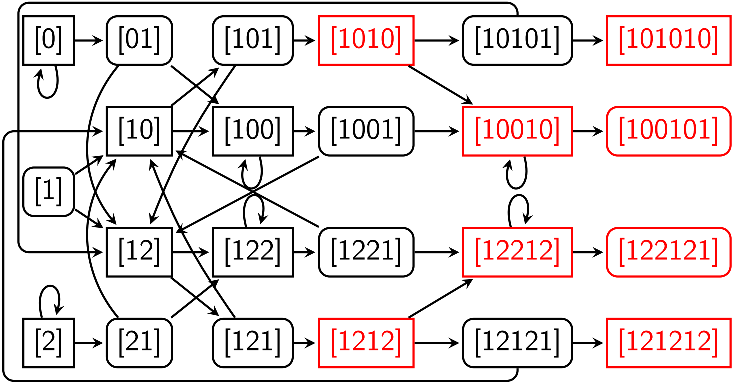

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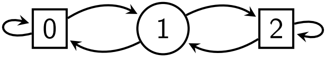

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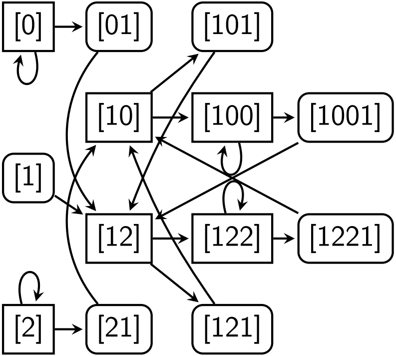
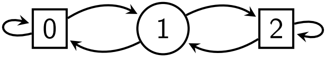
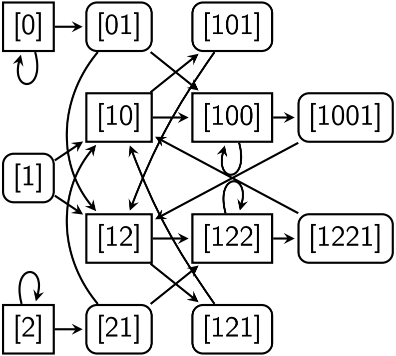


Use the winning region of safety game as memory structure..

The Proof: Safety to Muller





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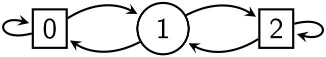
..or construct a permissive strategy..

The Proof: Safety to Muller



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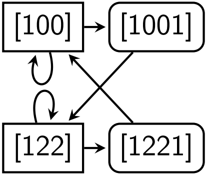
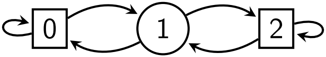


..or keep only maximal elements.

The Proof: Muller to Safety



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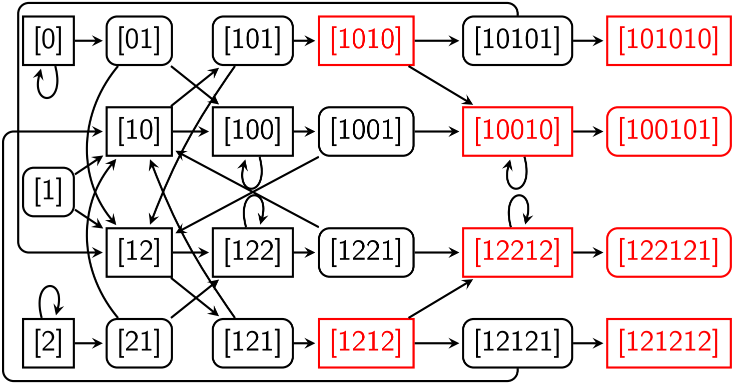

Mimic strategy that prevents Player 1 from reaching a score of 3.

Outline

1. Scoring Functions for Muller Games
2. Solving Muller Games by Solving Safety Games
3. **Conclusion**

Conclusion

Solving Muller games via safety games:

- New algorithm for Muller games: just solve the safety game.
- New memory structure for Muller games: maximal elements of winning region (antichain).
- New concept: permissive strategies for Muller games.
- Same constructions applicable for many other types of games.

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Ongoing and future work:

- A progress measure algorithm for Muller games?
- Is there a tradeoff between size and quality of a strategy?
- Can you play infinite games in infinite arenas in finite time?