Delay Games with WMSO+U Winning Conditions

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Max-Automata

- Deterministic finite automata with counters
- counter actions: `incr`, `reset`, `max`
- acceptance: boolean combination of "counter γ is bounded"

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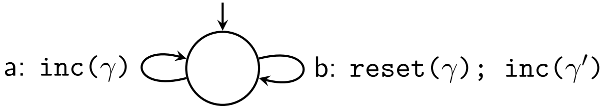

Acceptance condition: γ and γ' unbounded.

$$L(\mathcal{A}) = \{a^{n_0} b a^{n_1} b a^{n_2} b \cdots \mid \sup_i n_i = \infty\}$$

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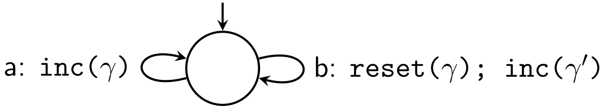

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Such automata capture weak MSO with the unbounding quantifier and thus many quantitative specification formalisms, e.g., finitary parity and Prompt-LTL.

Example

- $\Sigma_I = \{0, 1, \#\}$ and $\Sigma_O = \{0, 1, *\}$.
- Input block: $\#w$ with $w \in \{0, 1\}^+$.
- Output block:

$$\binom{\#}{\alpha(n)}\binom{\alpha(1)}{*}\binom{\alpha(2)}{*}\cdots\binom{\alpha(n-1)}{*}\binom{\alpha(n)}{\alpha(n)}$$

for $\alpha(j) \in \{0, 1\}$.

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Define language L_0: if infinitely many $\#$ and arbitrarily long input blocks, then arbitrarily long output blocks.

O wins with unbounded lookahead:

- If I produces arbitrarily long input blocks, then the lookahead will contain arbitrarily long input blocks.
- Thus, O can produce arbitrarily long output blocks.

Max-regular Delay Games

Theorem

Delay Games with max-regular winning conditions w.r.t fixed delay functions are determined.

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The following problem is decidable: given a max-automaton $\mathcal{A}$, does O win the delay game with winning condition $L(\mathcal{A})$ with bounded lookahead?

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Delay Games with max-regular winning conditions w.r.t fixed delay functions are determined.

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The following problem is decidable: given a max-automaton $\mathcal{A}$, does O win the delay game with winning condition $L(\mathcal{A})$ with bounded lookahead?

But: bounded lookahead is not always sufficient!

Bounded Lookahead is not Sufficient

Recall: O wins with unbounded lookahead.

- Input block: $\#w$ with $w \in \{0,1\}^+$.
- Output block: $\binom{\#}{\alpha(n)}\binom{\alpha(1)}{*}\binom{\alpha(2)}{*}\cdots\binom{\alpha(n-1)}{*}\binom{\alpha(n)}{\alpha(n)}$
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Claim: I wins with bounded lookahead:

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I: $\#$ 0 0 $\cdots$ 0

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- Lookahead contains only input blocks of bounded length.
- I can react to O's declaration at beginning of an output block to bound size of output blocks while producing arbitrarily large input blocks.

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If O wins a max-regular delay game with unbounded lookahead, then it does not matter how slow or fast the lookahead grows.

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Conjecture

Max-regular delay games with unbounded lookahead are decidable.